

Factor to Reveal

Use equivalent forms to uncover zeros and maximum values.

HIGH SCHOOL • AI-A.SSE.3

AI-A.SSE.3 "Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression."

NAME _____

DATE _____

FACTORED FORM REVEALS ZEROS

Factoring rewrites a quadratic as a product of simpler expressions. A product equals 0 when at least one factor equals 0, so factored form makes the zeros easy to find.

WORKED EXAMPLE

Rewrite $x^2 + 11x + 24$ in factored form. Use the factored form to find the zeros.

1. Find two numbers whose product is 24 and whose sum is 11: 3 and 8.
2. Rewrite: $x^2 + 11x + 24 = (x + 3)(x + 8)$.
3. Set each factor equal to 0: $x + 3 = 0$ or $x + 8 = 0$.

Answer: $(x + 3)(x + 8)$; zeros: $x = -3$ and $x = -8$

PRACTICE 6 problems

1 Factor $x^2 + 13x + 42$. Then state the zeros.

ANSWER _____

2 Factor $2x^2 + 19x + 45$. Then state the zeros.

ANSWER _____

3 Factor $5x^2 - 35x$. Then state the zeros.

ANSWER _____

4 Factor $x^2 - 14x + 49$. Then state the zero and its multiplicity.

ANSWER _____

5 Factor $4x^2 - 25$. Then state the zeros.

ANSWER _____

6 Factor $5x^2 + 37x + 14$. Then state the zeros.

ANSWER _____

APPLY IT Show your work

- 1** At a school soccer match, a kicked ball has height $h(t) = -5t^2 + 30t + 35$, where h is in meters and t is in seconds. Rewrite the expression in factored form. When does the ball hit the ground?

WORK SPACE

ANSWER _____

- 2** An esports club models its profit with $P(x) = -2x^2 + 50x - 200$, where x is the number of teams registered. Rewrite $P(x)$ in factored form. At how many teams does the club break even?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Rewrite $R(x) = -2x^2 + 28x + 90$ in vertex form by completing the square. Then explain how your form reveals the maximum value of R and the x -value where it occurs.

WORK SPACE AND ANSWER

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Factor $x^2 + 13x + 42$. Then state the zeros.	$(x + 6)(x + 7)$; zeros: $x = -6$ and $x = -7$
2	Factor $2x^2 + 19x + 45$. Then state the zeros.	$(2x + 9)(x + 5)$; zeros: $x = -9/2$ and $x = -5$
3	Factor $5x^2 - 35x$. Then state the zeros.	$5x(x - 7)$; zeros: $x = 0$ and $x = 7$
4	Factor $x^2 - 14x + 49$. Then state the zero and its multiplicity.	$(x - 7)(x - 7)$, or $(x - 7)^2$; $x = 7$ is a double zero
5	Factor $4x^2 - 25$. Then state the zeros.	$(2x - 5)(2x + 5)$; zeros: $x = 5/2$ and $x = -5/2$
6	Factor $5x^2 + 37x + 14$. Then state the zeros.	$(5x + 2)(x + 7)$; zeros: $x = -2/5$ and $x = -7$

PART 2 • APPLY IT

- $h(t) = -5(t - 7)(t + 1)$; the ball hits the ground at 7 seconds.** Factor out -5 to get $-5(t^2 - 6t - 7)$, then factor the trinomial. Set each factor equal to 0 and use the nonnegative time.
- $P(x) = -2(x - 5)(x - 20)$; the club breaks even with 5 teams or 20 teams.** Factor out -2 and factor the resulting quadratic. Profit is zero at the values that make either factor equal to zero.

CHALLENGE

$R(x) = -2(x - 7)^2 + 188$. Because $(x - 7)^2$ is at least 0, $-2(x - 7)^2$ is at most 0. Therefore, the maximum value is 188 when $x = 7$.

Accept equivalent factored forms with factors in either order. For zeros, accept the same values in any order, and require the double-zero description for problem 4.