LESSON MAKER PRO

Equation Reasoning Lab

Practice, apply, and defend each algebra step.

HIGH SCHOOL · AII-A.REI.1.b

AII-A.REI.1.b "Explain each step when solving rational or radical equations as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution..."

NAME ______________________ DATE ______________

KEEP EQUALITY TRUE

Start by assuming the original equation has a solution. Each operation must be done to both equal quantities, and you must state any value that would make a denominator zero. Squaring can create an extra answer, so check every candidate in the original equation.

WORKED EXAMPLE

Solve sqrt(x+20) = 13. Show why each step is valid.

1. Assume the two sides are equal. Squaring both equal sides gives x+20 = 169.
2. Subtracting 20 from both equal sides gives x = 149.
3. Check in the original equation: sqrt(149+20) = sqrt(169) = 13.

Answer: x = 149

PRACTICE 6 problems

1 Solve sqrt(x+5) = 3. Explain why subtracting 5 after squaring preserves equality.

ANSWER ______________________

2 Solve sqrt(x+10) = x-2. Squaring gives two candidates. Check both candidates in the original equation and identify the solution.

ANSWER ______________________

3 Solve 3/(x-1) = 2. State the restricted value and explain why you may multiply by x-1.

ANSWER ______________________

4 Solve 2/(x+1) = x. State the value that is not allowed.

ANSWER ______________________

5 Solve sqrt(2x-1) = x-2. Check each candidate in the original equation.

ANSWER ______________________

6 Solve 1/(x-1) + 1/(x+1) = 1. State the excluded values and justify multiplying by x^2-1.

ANSWER ______________________

APPLY IT Show your work

1 In a game design club, a bonus-point rule is sqrt(x+3) = x-3, where x is a test score. Solve the rule. Show why your algebra steps preserve equality, then check all candidates in the original rule.

WORK SPACE

ANSWER ______________________

2 A 24-GB game update downloads at x GB per minute. Its download time is 24/x minutes, and installation adds 2 minutes. The total time is 5 minutes. Solve 24/x + 2 = 5, and explain why clearing the denominator is valid.

WORK SPACE

ANSWER ______________________

CHALLENGE One step up

A student says that both x = 2 and x = 7 solve sqrt(x+2) = x-4 because both values appear after squaring. Write a viable argument that explains why squaring is allowed, why it may produce an extra candidate, and which value actually solves the original equation.

WORK SPACE AND ANSWER

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Solve sqrt(x+5) = 3. Explain why subtracting 5 after squaring preserves equality.	**x = 4. If sqrt(x+5) = 3, then squaring both equal sides gives x+5 = 9. Subtracting 5 from both equal sides gives x = 4.**
2	Solve sqrt(x+10) = x-2. Squaring gives two candidates. Check both candidates in the original equation and identify the solution.	**x = 6 only. Squaring gives x+10 = $(x-2)^2$, so x^2-5x-6 = 0 and x = 6 or x = -1. For x = 6, sqrt(16) = 4 = 6-2. For x = -1, sqrt(9) = 3 ≠ -3.**
3	Solve 3/(x-1) = 2. State the restricted value and explain why you may multiply by x-1.	**x = 5/2, with x ≠ 1. Because x-1 is nonzero when x ≠ 1, multiplying both equal sides by x-1 gives 3 = 2(x-1), so 2x = 5.**
4	Solve 2/(x+1) = x. State the value that is not allowed.	**x = 1 or x = -2, with x ≠ -1. Multiplying by the nonzero quantity x+1 gives 2 = x(x+1), so x^2+x-2 = 0.**
5	Solve sqrt(2x-1) = x-2. Check each candidate in the original equation.	**x = 5 only. Squaring gives 2x-1 = $(x-2)^2$, so x^2-6x+5 = 0 and the candidates are 1 and 5. At x = 1, sqrt(1) = 1 ≠ -1. At x = 5, sqrt(9) = 3 = 3.**
6	Solve 1/(x-1) + 1/(x+1) = 1. State the excluded values and justify multiplying by x^2-1.	**x = 1+sqrt(2) or x = 1-sqrt(2), with x ≠ 1 and x ≠ -1. For allowed values, x^2-1 is nonzero, so multiplying both equal sides by it gives 2x = x^2-1. Thus x^2-2x-1 = 0.**

PART 2 · APPLY IT

1. **x = 6 only. The candidate x = 1 fails because sqrt(4) = 2 ≠ -2. The candidate x = 6 works because sqrt(9) = 3 = 6-3.** Assuming the original sides are equal, square both sides to get x+3 = $(x-3)^2$. Solve x^2-7x+6 = 0, then check x = 1 and x = 6 in the original rule.
2. **x = 8 GB per minute, with x ≠ 0. Since a download rate cannot be zero, multiplying both equal sides by the nonzero value x gives 24+2x = 5x. Then 24 = 3x, so x = 8.** State x ≠ 0 before multiplying both sides by x. Solve the resulting linear equation and verify that 24/8 + 2 = 5.

CHALLENGE

If the original sides are equal, then their squares are equal, so squaring is a valid step for finding candidates. It gives x+2 = $(x-4)^2$, so x^2-9x+14 = 0 and the candidates are x = 2 and x = 7. Squaring may create an extra candidate, so check both. For x = 2, sqrt(4) = 2 ≠ -2, so reject it. For x = 7, sqrt(9) = 3 = 7-4, so x = 7 is the solution.

Accept equivalent algebraic forms and clear statements that the same permitted operation is applied to equal quantities. For radical equations, require checks in the original equation and reject extraneous candidates. For rational equations, require excluded denominator values before clearing denominators.