

Form Reveals More

Rewrite quadratics to uncover zeros and extreme values.

HIGH SCHOOL • AII-A.SSE.3

AII-A.SSE.3 "Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression."

NAME _____

DATE _____

CHOOSE THE USEFUL FORM

An equivalent form has the same value for every x , but it can make a different feature easier to see. Factored form shows the zeros, and vertex form shows the maximum or minimum value. To make vertex form, factor the leading coefficient when needed, then complete the square.

WORKED EXAMPLE

Rewrite $3x^2 - 18x + 4$ in vertex form. State the minimum value.

- Factor 3 from the first two terms: $3(x^2 - 6x) + 4$.
- Complete the square: $3[(x - 3)^2 - 9] + 4$.
- Simplify: $3(x - 3)^2 - 23$.

Answer: $3(x - 3)^2 - 23$; the minimum value is -23 at $x = 3$.

PRACTICE 6 problems

1 Factor $x^2 + 13x + 40$. Then state its zeros.

ANSWER _____

2 Rewrite $x^2 - 14x + 41$ in vertex form. State the vertex.

ANSWER _____

3 Rewrite $5(x - 2)(x + 6)$ in standard form.

ANSWER _____

4 Factor $36x^2 - 49$. Then state its zeros.

ANSWER _____

5 Rewrite $2x^2 - 20x + 43$ in vertex form. State the minimum value.

ANSWER _____

6 Factor $6x^2 - 7x - 10$ completely.

ANSWER _____

APPLY IT Show your work

- 1** A student sells custom phone cases. The profit, in hundreds of dollars, is $-x^2 + 16x - 15$, where x is the number of cases sold in tens. Rewrite the expression in vertex form. What is the greatest possible profit, and how many cases must be sold?

WORK SPACE

ANSWER _____

- 2** During a video shoot, the height of a launched foam ball, in feet, is $-5x^2 + 30x$, where x is the number of seconds after launch. Factor the expression. When does the ball land after it is launched?

WORK SPACE

ANSWER _____

CHALLENGE One step up

For $2x^2 - 24x + 70$, write both a factored form and a vertex form. Use the forms to explain the zeros, the minimum value, and where the minimum occurs.

WORK SPACE AND ANSWER

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Factor $x^2 + 13x + 40$. Then state its zeros.	$(x + 5)(x + 8)$; zeros: -5 and -8
2	Rewrite $x^2 - 14x + 41$ in vertex form. State the vertex.	$(x - 7)^2 - 8$; vertex: (7, -8)
3	Rewrite $5(x - 2)(x + 6)$ in standard form.	$5x^2 + 20x - 60$
4	Factor $36x^2 - 49$. Then state its zeros.	$(6x - 7)(6x + 7)$; zeros: $7/6$ and $-7/6$
5	Rewrite $2x^2 - 20x + 43$ in vertex form. State the minimum value.	$2(x - 5)^2 - 7$; the minimum value is -7 at $x = 5$
6	Factor $6x^2 - 7x - 10$ completely.	$(6x + 5)(x - 2)$

PART 2 • APPLY IT

- $-(x - 8)^2 + 49$; greatest profit: \$4,900, when 80 cases are sold** Complete the square to get $-(x - 8)^2 + 49$. The vertex gives 49 hundreds of dollars when $x = 8$, or 80 cases.
- $-5x(x - 6)$; the ball lands 6 seconds after launch** Factor out $-5x$ to get $-5x(x - 6)$. The zeros are $x = 0$ and $x = 6$, and the positive time is the landing time.

CHALLENGE

Factored form: $2(x - 5)(x - 7)$. Vertex form: $2(x - 6)^2 - 2$. The zeros are 5 and 7. The minimum value is -2, and it occurs at $x = 6$.

Accept equivalent forms with factors in either order and equivalent vertex forms. For contextual questions, require both the numerical result and the correct units.