

Forms Reveal More

Rewrite quadratic functions to show zeros, vertices, and maximum or minimum values.

HIGH SCHOOL • AII-F.IF.8

AII-F.IF.8 "Write a function in different but equivalent forms to reveal and explain different properties of the function."

NAME _____

DATE _____

CHOOSE THE HELPFUL FORM

Equivalent forms name the same function, but each form makes a different property easy to see. Factored form shows zeros, vertex form shows the vertex and maximum or minimum, and standard form shows the y-intercept. You can rewrite a quadratic to reveal the property you need.

WORKED EXAMPLE

Write $f(x) = x^2 + 14x + 40$ in vertex form and factored form. Then state the vertex and zeros.

1. $f(x) = x^2 + 14x + 49 - 49 + 40$

2. $f(x) = (x + 7)^2 - 9$

3. $x^2 + 14x + 40 = (x + 4)(x + 10)$

4. The vertex is $(-7, -9)$, and the zeros are $x = -4$ and $x = -10$.

Answer: Vertex form: $f(x) = (x + 7)^2 - 9$.

Factored form: $f(x) = (x + 4)(x + 10)$.

Vertex: $(-7, -9)$. Zeros: -4 and -10 .

PRACTICE 6 problems

- 1** Write $g(x) = x^2 - 8x + 12$ in vertex form. Then state its vertex.

ANSWER _____

- 2** Write $h(x) = (x + 5)^2 - 9$ in standard form. Then find its zeros.

ANSWER _____

- 3** Write $p(x) = (x - 2)(x + 9)$ in standard form. Then state its y-intercept.

ANSWER _____

- 4** Write $q(x) = x^2 + x - 20$ in factored form. Then state its zeros.

ANSWER _____

- 5** Write $r(x) = -3(x - 1)^2 + 12$ in standard form. Then state its maximum value and where it occurs.

ANSWER _____

- 6** Write $s(x) = 2(x + 3)(x - 5)$ in vertex form. Then state its axis of symmetry.

ANSWER _____

APPLY IT Show your work

- 1** A skateboarder's height above the ground, in feet, after x seconds is modeled by $h(x) = -2x^2 + 12x + 5$. Write the function in vertex form. What is the skateboarder's greatest height, and after how many seconds does it occur?

WORK SPACE

ANSWER _____

- 2** A student club models its profit from a concert ticket price of x dollars with $P(x) = -2x^2 + 24x - 64$. Write $P(x)$ in factored form. What ticket prices give a profit of 0 dollars?

WORK SPACE

ANSWER _____

CHALLENGE One step up

A quadratic function has vertex $(5, -12)$ and zeros $x = 3$ and $x = 7$. Write the function in vertex form, factored form, and standard form. Explain how the zeros and vertex determine the leading coefficient.

WORK SPACE AND ANSWER

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Write $g(x) = x^2 - 8x + 12$ in vertex form. Then state its vertex.	$g(x) = (x - 4)^2 - 4$; vertex: (4, -4)
2	Write $h(x) = (x + 5)^2 - 9$ in standard form. Then find its zeros.	$h(x) = x^2 + 10x + 16$; zeros: $x = -8$ and $x = -2$
3	Write $p(x) = (x - 2)(x + 9)$ in standard form. Then state its y-intercept.	$p(x) = x^2 + 7x - 18$; y-intercept: (0, -18)
4	Write $q(x) = x^2 + x - 20$ in factored form. Then state its zeros.	$q(x) = (x + 5)(x - 4)$; zeros: $x = -5$ and $x = 4$
5	Write $r(x) = -3(x - 1)^2 + 12$ in standard form. Then state its maximum value and where it occurs.	$r(x) = -3x^2 + 6x + 9$; maximum value: 12 at $x = 1$
6	Write $s(x) = 2(x + 3)(x - 5)$ in vertex form. Then state its axis of symmetry.	$s(x) = 2(x - 1)^2 - 32$; axis of symmetry: $x = 1$

PART 2 • APPLY IT

1. $h(x) = -2(x - 3)^2 + 23$. The greatest height is 23 feet, and it occurs after 3 seconds. Complete the square after factoring out -2 from the quadratic and linear terms. The vertex form shows the maximum because the leading coefficient is negative.
2. $P(x) = -2(x - 4)(x - 8)$. A profit of 0 dollars occurs when tickets cost \$4 or \$8. Factor out -2, then factor $x^2 - 12x + 32$. Set each factor equal to 0 to find the break-even ticket prices.

CHALLENGE

Vertex form: $f(x) = 3(x - 5)^2 - 12$. **Factored form:** $f(x) = 3(x - 3)(x - 7)$. **Standard form:** $f(x) = 3x^2 - 30x + 63$. Since $x = 3$ is a zero, $0 = a(3 - 5)^2 - 12 = 4a - 12$, so $a = 3$.

Accept equivalent forms with terms written in a different order. For vertices, accept either ordered-pair notation or a statement giving the x-value and y-value.