

Unit Circle Paths

Connect arcs, angles, and radians on a unit circle.

HIGH SCHOOL • AII-F.TF.1

AII-F.TF.1 "Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle."

NAME _____

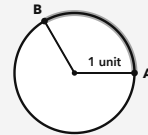
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ARC LENGTH EQUALS RADIAN

On a unit circle, the radius is 1, so the length of a subtended arc is equal to the angle's radian measure. To change degrees to radians, multiply the degree measure by $\pi/180$.

WORKED EXAMPLE

Use the unit circle shown. Central angle AOB measures 120° . Find its radian measure and the length of its highlighted arc.

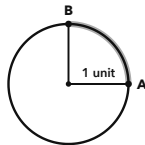


1. Convert the angle: $120^\circ \times \pi/180^\circ = 2\pi/3$ radians.
2. Use $s = r\theta$ with $r = 1$.
3. $s = 1(2\pi/3) = 2\pi/3$ units.

Answer: $2\pi/3$ radians; $2\pi/3$ units

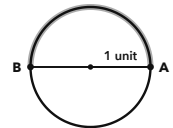
PRACTICE 6 problems

- 1** Use the unit circle shown. Find the radian measure of central angle AOB.



ANSWER _____

- 2** Use the unit circle shown. Find the length of the highlighted arc from A to B.



ANSWER _____

- 3** A unit-circle arc subtends a central angle of 45° . What is the arc length?

ANSWER _____

- 4** A central angle on a unit circle measures 270° . What is its radian measure?

ANSWER _____

- 5** On a unit circle, an arc has length $5\pi/6$ units. What is the measure of its central angle in radians?

ANSWER _____

- 6** A central angle on a unit circle measures $7\pi/6$ radians. What is the length of its subtended arc?

ANSWER _____

APPLY IT Show your work

1

A music app animates a marker moving counterclockwise around a unit-circle album-art ring. During one transition, the marker turns 210° . What is its radian measure, and how far does the marker travel along the ring?

WORK SPACE

ANSWER _____

2

A fitness app uses a unit-circle progress ring. A workout fills 135° of the ring. Find the radian measure of the filled angle and the length of the filled arc.

WORK SPACE

ANSWER _____

CHALLENGE One step up

In an animation, a dot begins at angle 0 on a unit circle and moves counterclockwise through $13\pi/6$ radians. How far does the dot travel along the circle? Give the dot's final position as its least nonnegative radian measure. Explain why the travel distance is not $\pi/6$.

WORK SPACE AND ANSWER

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Use the unit circle shown. Find the radian measure of central angle AOB.	$\pi/2$ radians
2	Use the unit circle shown. Find the length of the highlighted arc from A to B.	π units
3	A unit-circle arc subtends a central angle of 45° . What is the arc length?	$\pi/4$ units
4	A central angle on a unit circle measures 270° . What is its radian measure?	$3\pi/2$ radians
5	On a unit circle, an arc has length $5\pi/6$ units. What is the measure of its central angle in radians?	$5\pi/6$ radians
6	A central angle on a unit circle measures $7\pi/6$ radians. What is the length of its subtended arc?	$7\pi/6$ units

PART 2 • APPLY IT

- $7\pi/6$ radians; $7\pi/6$ units** Convert 210° by multiplying by $\pi/180^\circ$ to get $7\pi/6$ radians. Because the ring is a unit circle, the arc length is also $7\pi/6$ units.
- $3\pi/4$ radians; $3\pi/4$ units** Convert 135° to radians: $135^\circ \times \pi/180^\circ = 3\pi/4$. On a unit circle, that same value is the arc length.

CHALLENGE

The dot travels $13\pi/6$ units and ends at $\pi/6$ radians. Since $13\pi/6 = 2\pi + \pi/6$, the dot makes one full turn plus $\pi/6$, so its final location is $\pi/6$ but its path includes the full turn.

Accept equivalent exact forms, such as $\pi/2$ units for an arc length. Require radians for angle measures and units for arc lengths when requested.