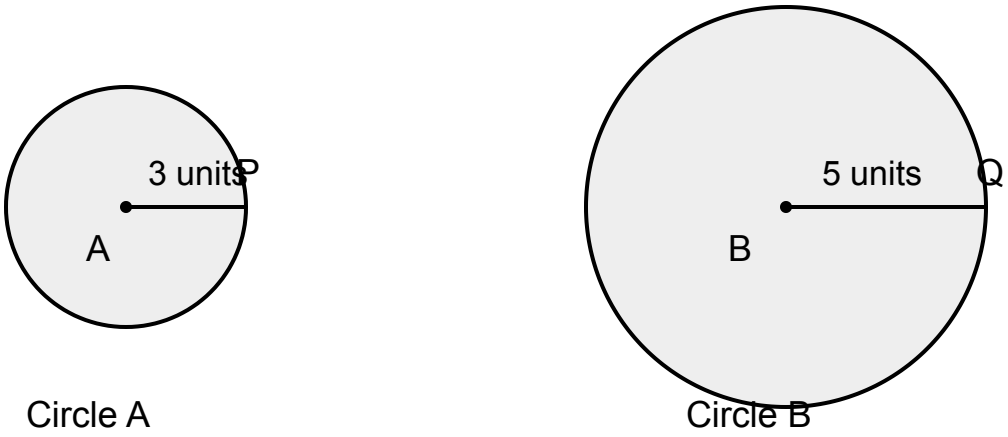
Mapping One Circle to Another

Use a translation and a dilation to prove all circles are similar

Name: _____ Date: _____

Part 1 - Build a mapping

Goal: Map Circle A to Circle B using a translation followed by a dilation. A circle's size and shape are determined by its radius; its position is determined by its center.



1. First translate Circle A so that its center A lands on B. Then dilate the translated circle with center B. What scale factor maps the 3-unit radius to the 5-unit radius?

2. In this workspace, sketch the translated circle and the final dilated circle. Mark the image of P after each step.

3. Explain why the final circle is Circle B, not merely a circle with the same radius.

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Part 2 - Generalize the proof

Let Circle C have center C and positive radius r. Let Circle D have center D and positive radius R.

4. Complete the transformation rule.

Step 1: Translate so that C maps to _____.

Step 2: Dilate with center _____ and scale factor _____.

5. Let P be any point on Circle C. After the translation, its image P1 is how far from D? After the dilation, its image P2 is how far from D? Show the calculation.

6. Use your work to write a proof that every point of Circle C maps onto Circle D. Then state why this proves the circles are similar. Use r > 0 and R > 0 in your reasoning.

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Answer key and teaching notes

Materials and setup: Print and supply all student pages, keeping teacher-key pages separate. Students need pencils, rulers, and optional tracing paper or compass. Display or distribute the task without adding a premade transformation diagram. Students may draw freehand in the workspace; accurate construction tools are optional.

Teaching script: Say: “A translation changes location but not distances. A dilation multiplies every distance from its center by one scale factor. Today, we will map the center and every radius, not just compare two radius numbers.” Ask students to listen, then read Part 1 directions aloud. After individual work, say: “Choose any point on the first circle. What must happen to its distance from the new center?” Do not tell students the general scale factor before Part 2.

Observe: While students work, check whether each student (1) translates A to B, (2) uses dilation center B, (3) calculates 5/3, and (4) explains both the center and radius condition. For Part 2, listen for the statement that an arbitrary point at distance r becomes a point at distance R. Prompt only: “Where is the dilation centered?”

1. Translate Circle A by vector AB, so A maps to B. Dilate about B by scale factor 5/3.

2. A valid sketch shows an intermediate circle centered at B with radius 3 units, then a final circle centered at B with radius 5 units. P first translates to P1, a point 3 units from B; dilation sends P1 to P2, 5 units from B. Labels may vary.

3. The translated-and-dilated circle has center B. Its radius is (5/3)(3) = 5 units. A circle with center B and radius 5 units is exactly Circle B.

4. Translate so C maps to D. Dilate with center D and scale factor R/r.

5. After translation, DP1 = r because translations preserve distance. After dilation, DP2 = (R/r)r = R.

6. Sample proof: Translate Circle C so its center C maps to D. Translation preserves distances, so every point P on Circle C maps to P1 with DP1 = r. Because r and R are positive, R/r is a positive valid scale factor. Dilate about D by R/r. Then DP2 = (R/r)r = R. Thus every image point lies on Circle D. Conversely, a point at distance R from D has a preimage at distance r under the inverse dilation and inverse translation, so the entire Circle D is the image. The composition of a translation and a dilation maps one circle onto the other; therefore all circles are similar.

Source credit: New York Next Generation Mathematics Learning Standards, GEO-G.C.1, “Prove that all circles are similar.” https://lessonmakerpro.com/standards/ny-nextgen-math/geo-g-c-1