Polygons Inside a Circle

Construct and justify an inscribed regular hexagon and equilateral triangle

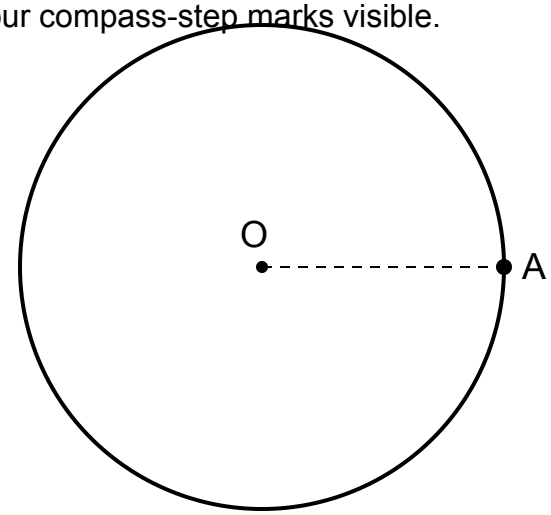
Name: _____ Date: _____

Part 1 - Inscribe a Regular Hexagon

Tools: compass, straightedge, and pencil. Do not erase compass marks.

1. In the circle below, point A is on the circle and O is the center. Open your compass from O to A. Keep that opening unchanged. Starting at A, step the compass around the circle and mark five new points. Use a straightedge to connect consecutive points. You should make a 6-sided polygon.

Construction area - leave your compass-step marks visible.



2. Check your construction. Are the six short arcs between neighboring vertices equal? Are the six sides equal? Explain what you observed.

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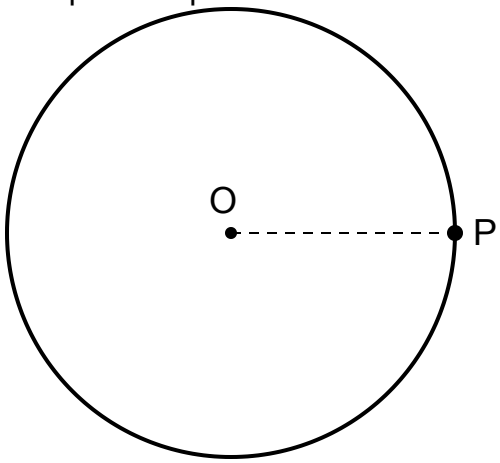
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Part 2 - Inscribe an Equilateral Triangle and Explain

3. Use the same compass idea in this new circle. Set the compass opening from O to P. Step around the circle to make six equally spaced marks. Then connect every other mark, starting at P, to make a triangle. Leave all six marks and compass arcs visible.

Construction area - leave your compass-step marks visible.



4. Justify the hexagon: A full circle is 360 degrees. What is the central angle for each of 6 equal steps? Why does that make the hexagon regular?

5. Justify the triangle: Your triangle uses every other one of the six marks. What is the central angle between neighboring triangle vertices? Why are all three triangle sides equal?

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Answer Key and Teaching Notes

Materials: Print and supply all student pages, keeping teacher-key pages separate. Each student needs a sharpened pencil, compass that holds its opening, and straightedge. Optional: a light-colored pencil for construction arcs. Do not provide a protractor for the construction.

Setup: Before class, test-print the circles at full size. Ask students to place a compass point precisely on a marked point and to keep the pencil point on the circle while stepping. Tell students that visible arcs, dots, and straightedge segments are evidence of construction, not messy work.

Teacher-read script: “A construction is not tracing a finished shape. Your compass marks show how you located the vertices. Keep the same compass opening while you step around each circle. After you construct, use equal arcs and central angles to explain why the polygon has equal sides.”

Observation method: During Parts 1 and 2, use this checklist for each student: (a) compass opening matches radius OA or OP; (b) student makes six positions around the circle rather than estimating vertices; (c) construction marks remain visible; (d) straightedge is used for sides; (e) student connects consecutive points for the hexagon and alternating points for the triangle. Confer with a student who has a gap or overlap after stepping: check compass opening and placement before allowing a restart.

1. Regular hexagon construction: The student should make five additional points by repeatedly stepping a compass opening equal to radius OA around the circle, then connect all six consecutive points. Each side is one radius-length chord. Accept a rotated or reversed construction.

2. Check: Accept: “The six stepped arcs/chords match because the compass opening did not change.” More complete reasoning notes that each adjacent pair of radii and the chord between them form congruent equilateral triangles, so the six central angles and six sides are equal.

3. Equilateral triangle construction: The student first creates six equally spaced circle points, then connects P to the point two steps away, then to the next alternating point, and back to P. The completed triangle has three vertices among the six hexagon positions. Construction dots/arcs should remain visible.

4. Hexagon justification: 360 degrees divided by 6 equal steps is 60 degrees per central angle. Equal central angles in the same circle intercept equal arcs and equal chords. Therefore all six sides are equal; with six equal central angles, the inscribed polygon is a regular hexagon.

5. Triangle justification: Each triangle vertex is separated by two 60-degree steps, so each central angle is 120 degrees. The three central angles are equal, so they intercept equal arcs and equal chords. Therefore the three sides are equal, making an equilateral triangle. Accept equivalent congruent-triangle reasoning.

Standard: New York GEO-G.CO.13. This task practices inscribed hexagons and equilateral triangles. https://lessonmakerpro.com/standards/ny-nextgen-math/geo-g-co-13