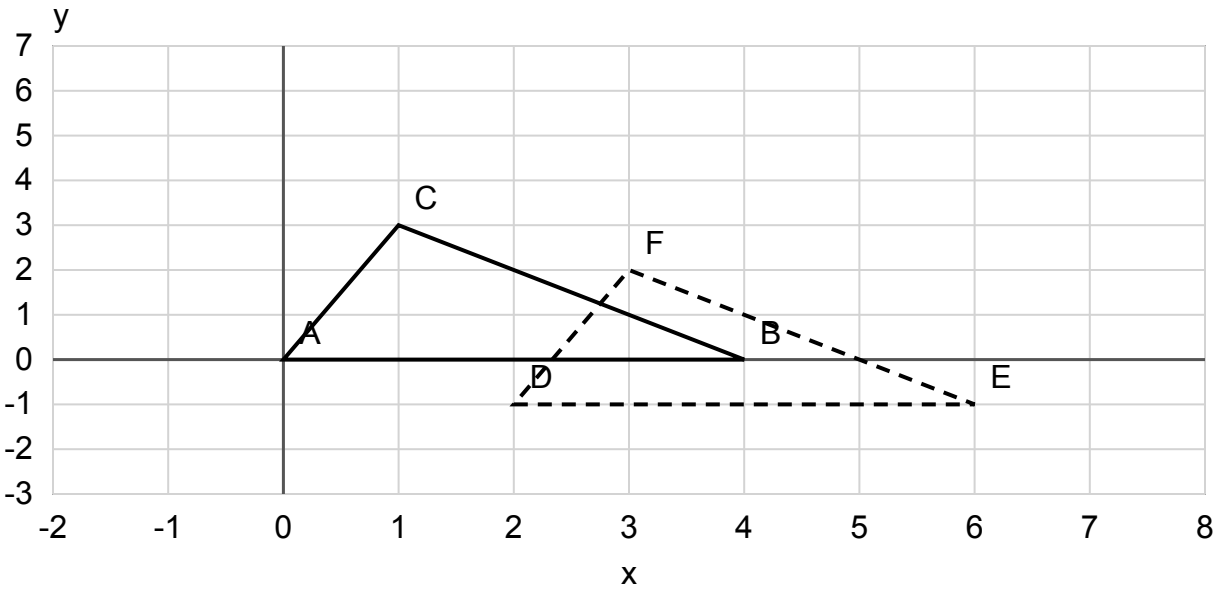
Rigid Motions and Triangle Congruence

Map corresponding vertices and justify both directions of triangle congruence

Name: _____ Date: _____

Part 1 - Map the triangles

Coordinate model. Triangle ABC is drawn with solid sides. Triangle DEF is drawn with dashed sides. The stated correspondence is A to D, B to E, and C to F.



1. What translation maps ABC onto DEF? Write a coordinate rule and a vector.

2. Verify the map by finding AB and DE. Then explain why every corresponding side length and every corresponding angle measure agrees after this translation.

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Part 2 - Why the converse is true

Congruence statement: Two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent.

For triangles PQR and XYZ, suppose PQ = XY, QR = YZ, PR = XZ, and all three corresponding angle pairs are equal. The correspondence is P to X, Q to Y, R to Z.

3. Forward direction. Complete this idea: If a rigid motion maps PQR onto XYZ, then the triangles have matching side lengths and angle measures because rigid motions...

4. Converse direction. Explain how to build a rigid motion that maps P to X and Q to Y. Then explain what can be done if R and Z land on opposite sides of line XY. Why does this place R on Z?

5. Finish the statement in your own words: Matching corresponding sides and angles is enough to conclude that...

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Answer key and teaching notes

Materials and setup: Print and supply all student pages, keeping teacher-key pages separate. Supply pencils, rulers, and optional tracing paper. Students need no calculator. Before students begin Part 1, point out that solid and dashed outlines distinguish the two triangles; color is not needed.

Teaching script: Say: “A rigid motion can slide, turn, or flip a figure without stretching it. Today you will use a slide, then explain why matching measures also guarantee a rigid-motion match.” Say: “Corresponding vertices are named for you. Follow A to D, B to E, and C to F.” Before Part 2, say: “For the converse, imagine first placing one side exactly on its matching side. A possible flip is still a rigid motion.” Do not tell students the coordinate rule before they attempt item 1.

Observation method: As students work, check whether each student (a) pairs A-D, B-E, and C-F; (b) identifies the same horizontal and vertical change for all vertices; and (c) names preservation of distance and angle measure. During Part 2, listen for a placement of one side followed by a reflection option. Prompt only with: “Where could a point be if its distances to both endpoints are fixed?” Record each criterion as demonstrated, developing, or not yet demonstrated.

1. Translation rule: (x, y) to (x + 2, y - 1), or translation vector <2, -1>. A goes (0, 0) to (2, -1); B goes (4, 0) to (6, -1); C goes (1, 3) to (3, 2).

2. AB = 4 and DE = 4. A translation is a rigid motion, so it preserves distances and angle measures. Therefore AB = DE, AC = DF, BC = EF, and all corresponding angles have equal measures. Accept a correct distance calculation such as AB = sqrt((4 - 0)^2 + (0 - 0)^2) = 4 and DE = sqrt((6 - 2)^2 + (-1 - (-1))^2) = 4.

3. Rigid motions preserve distance and angle measure. Therefore corresponding sides and angles match. Accept “do not stretch or change angle openings” as an age-appropriate equivalent.

4. One valid explanation: Use translations and rotations to send P to X and Q to Y, so segment PQ lies on segment XY with the same length. Point R must be at distances PR = XZ from X and QR = YZ from Y. Thus it is located at an intersection of circles centered at X and Y with those radii. If the placed R is on the opposite side of XY from Z, reflect across line XY. This keeps X and Y fixed and moves R to the matching intersection, Z. The angle information is also consistent with this correspondence; matching three sides alone already determines the placement up to a reflection.

5. Sample response: If corresponding sides and angles match, a sequence of rigid motions can map one triangle onto the other, so the triangles are congruent. Require both the rigid-motion conclusion and the word “congruent” for full credit.

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