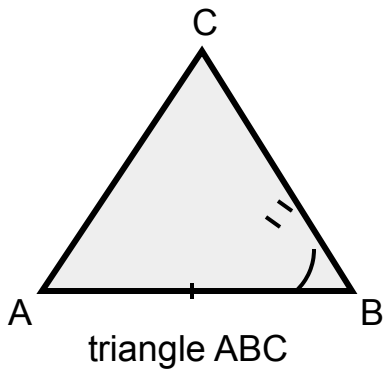
Rigid Motions Prove Triangle Congruence

Explain why SAS and ASA fix a triangle, and why SSA may not

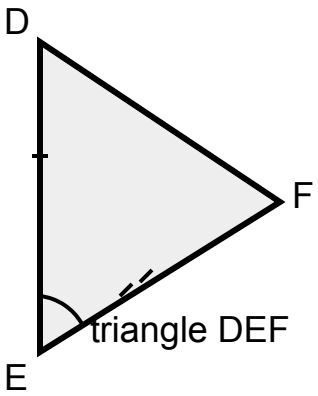
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Part 1 - SAS: Build the alignment

Rigid-motion idea: A translation, rotation, or reflection preserves lengths and angle measures. For SAS, two side lengths and the angle *between* those sides fix the location of the third vertex.



triangle ABC



triangle DEF

1. The tick marks and arcs are the givens. Complete the correspondence: A maps to ___, B maps to ___, and C maps to ___. Then write a motion sequence that places triangle ABC exactly on triangle DEF. You may name a rotation and a translation, or another valid sequence.

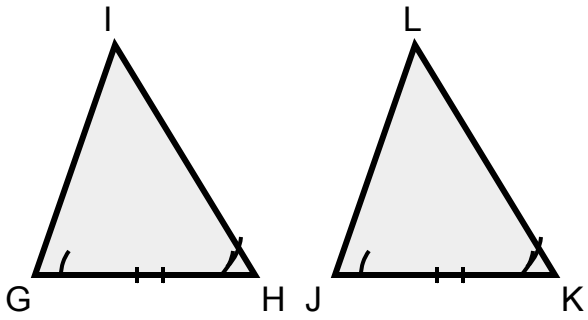
2. Why does the marked information force C to land on F after the two marked sides and included angle are aligned? Explain why there is not a second possible location on the chosen side of the base.

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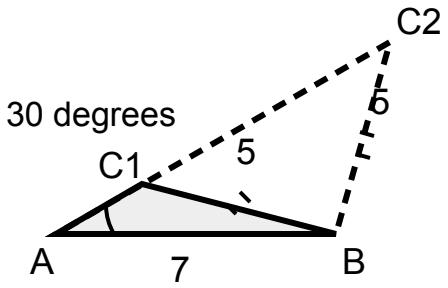
Part 2 - ASA works; SSA can branch



3. ASA

The single arcs show angle G = angle J. The double arcs show angle H = angle K. The double ticks show GH = JK.

Describe how to align GH with JK. Then explain why the two angle rays force I to land on L. Include the word **intersection**.



Both triangles have AB = 7, a 30-degree angle at A, and BC = 5.

4. SSA contrast. The solid and dashed triangles have the same two side lengths and a nonincluded angle. Explain how C1 and C2 can both satisfy the givens. Therefore, why cannot SSA alone prove the triangles congruent?

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Answer key and teaching notes

Materials and setup: Print and supply all student pages, keeping teacher-key pages separate. Supplied: marked triangle diagrams and writing organizers. Additionally needed: pencil; optional ruler and tracing paper. Before students begin Part 1, have them identify the matching tick marks aloud. Do not tell them the criterion names until they have described the alignment.

Teaching script: Say: “A rigid motion does not stretch a figure. Our job is not only to name a test; it is to explain why the marked parts leave no freedom for the last vertex. As you write, imagine moving one triangle onto the other.” For Part 2 say: “A point where two already-placed rays meet has a forced location. But a circle and a ray can sometimes meet twice.”

Observation method: While students work, listen for a motion, correct vertex correspondence, and a reason that the final vertex is forced. Mark each item: secure if all three appear; developing if a criterion is named but the rigid-motion reason is missing; reteach if the student treats nonincluded SSA as SAS. Ask a developing student: “Which two rays or distances locate the last point?”

1. Answer: A maps to D, B maps to E, and C maps to F. One valid sequence is to rotate triangle ABC 90 degrees clockwise about B, then translate the image so B lands on E. Other valid rigid-motion sequences are acceptable if all corresponding vertices align. The single-tick sides AB and DE align, the double-tick sides BC and EF align, and the included angle at B aligns with the angle at E.

2. Answer: Once the two sides and their included angle are aligned, the ray for the second side has one fixed direction and fixed length. Its endpoint is forced to F. A different endpoint would change a marked side length or the included angle. This explains SAS from rigid motions, not merely its name.

3. Answer: Move GH onto JK by a translation and rotation; reflect if needed so the triangle is on the matching side. With G on J and H on K, the ray from J making the single-arc angle and the ray from K making the double-arc angle are fixed. Their intersection is L, so I must land on L. Thus ASA fixes the triangle.

4. Answer: C1 and C2 are different points on the same 30-degree ray from A, and each is 5 units from B. Thus each can meet AB = 7, angle A = 30 degrees, and BC = 5, yet the triangles have different third-side lengths and shapes. SSA does not always determine a unique triangle, so it is insufficient for a congruence proof.

Source credit: New York Next Generation Mathematics, GEO-G.CO.8, teacher-provided standard record: https://lessonmakerpro.com/standards/ny-nextgen-math/geo-g-co-8