LESSON MAKER PRO

Reverse the Model

Find inverse rules and graph reversed input-output pairs.

HIGH SCHOOL · A2.F.2.3

A2.F.2.3 "Find and graph the inverse of a function, if it exists, in mathematical models. Know that the domain of a function f is the range of the inverse function f^{-1} and the range of the function f is the domain of the inverse function f^{-1}."

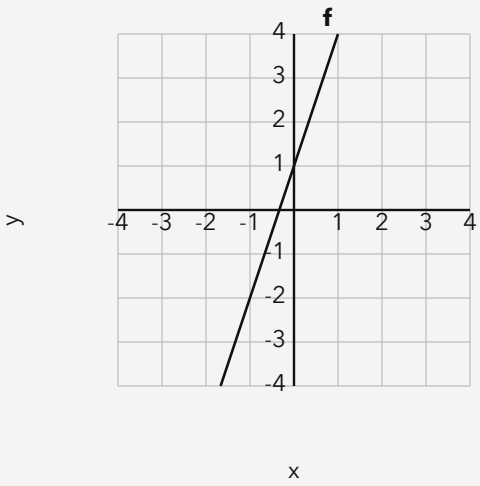
NAME _____ DATE _____

SWITCH AND REFLECT

An inverse function switches the inputs and outputs of a one-to-one function. On a graph, each point (a, b) becomes (b, a), so the original domain becomes the inverse range and the original range becomes the inverse domain.

WORKED EXAMPLE

The grid shows f. Find $f^{-1}(x)$ and graph it on the same grid.

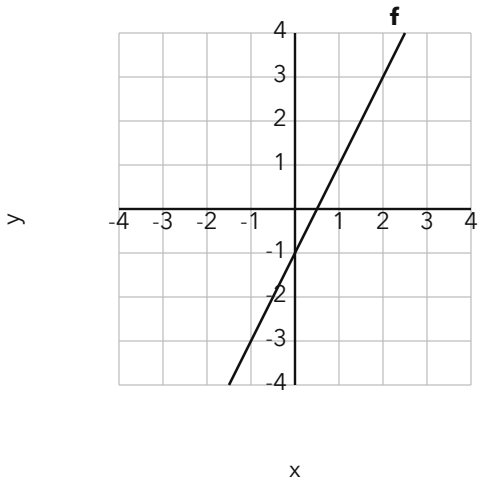


1. Read two points on f: (0, 1) and (1, 4).
2. Switch the coordinates to get (1, 0) and (4, 1) on f^{-1}.
3. The new line has slope 1/3 and passes through (1, 0), so $f^{-1}(x) = (x - 1)/3$.

Answer: $f^{-1}(x) = (x - 1)/3$; graph the line through (1, 0) and (4, 1).

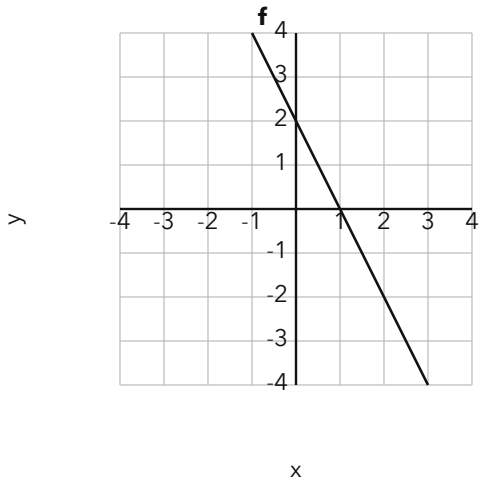
PRACTICE 6 problems

1 Graph f^{-1} on the grid and write its equation.



ANSWER _____

2 Graph f^{-1} on the grid and write its equation.



ANSWER _____

3 For $f(x) = 4x - 5$, find $f^{-1}(x)$. State the domain of f^{-1}.

ANSWER _____

4 For $f(x) = (x - 6)/5$ with domain $x \geq 6$, find $f^{-1}(x)$. State its domain and range.

ANSWER _____

5 For $f(x) = x^2$ with domain $x \geq 0$, find $f^{-1}(x)$ and state its domain.

ANSWER _____

6 Does $f(x) = x^2 - 4$, with domain all real numbers, have an inverse function? Explain using two inputs.

ANSWER _____

APPLY IT Show your work

1 A rideshare fare is modeled by $C(m) = 2m + 6$, where m is miles traveled and $m \geq 0$. Find $C^{-1}(x)$, state its domain, and use it to find the miles traveled for a $16 fare.

WORK SPACE

ANSWER _____

2 A phone's battery level is modeled by $B(t) = 80 - 8t$, where B is the battery percent and $0 \leq t \leq 10$ hours. Find $B^{-1}(x)$, state its domain and range, and use it to find when the battery reaches 32%.

WORK SPACE

ANSWER _____

CHALLENGE One step up

For $f(x) = (2x - 3)/(x + 4)$, find $f^{-1}(x)$. Which input is excluded from the domain of f, and which input is excluded from the domain of f^{-1}? Explain how the exclusions are related.

WORK SPACE AND ANSWER

ANSWER KEY

Reverse the Model

HIGH SCHOOL · A2.F.2.3

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Graph f^{-1} on the grid and write its equation.	**$f^{-1}(x) = (x + 1)/2$; graph the line through (-1, 0), (1, 1), and (3, 2).**
2	Graph f^{-1} on the grid and write its equation.	**$f^{-1}(x) = 1 - x/2$; graph the line through (-2, 2), (0, 1), and (2, 0).**
3	For $f(x) = 4x - 5$, find $f^{-1}(x)$. State the domain of f^{-1}.	**$f^{-1}(x) = (x + 5)/4$; domain: all real numbers.**
4	For $f(x) = (x - 6)/5$ with domain $x \geq 6$, find $f^{-1}(x)$. State its domain and range.	**$f^{-1}(x) = 5x + 6$; domain: $x \geq 0$; range: $y \geq 6$.**
5	For $f(x) = x^2$ with domain $x \geq 0$, find $f^{-1}(x)$ and state its domain.	**$f^{-1}(x) = \text{sqrt}(x)$; domain: $x \geq 0$.**
6	Does $f(x) = x^2 - 4$, with domain all real numbers, have an inverse function? Explain using two inputs.	**No. Both f(-1) and f(1) equal -3, so an inverse would have to send -3 to two outputs.**

PART 2 · APPLY IT

1. **$C^{-1}(x) = (x - 6)/2$; domain: $x \geq 6$; a $16 fare represents 5 miles.** Switch the fare and miles, then solve for miles. The smallest possible fare is $6, and $C^{-1}(16) = 5$.
2. **$B^{-1}(x) = (80 - x)/8$; domain: $0 \leq x \leq 80$; range: $0 \leq y \leq 10$; the battery reaches 32% after 6 hours.** Solve $x = 80 - 8t$ for t. The inverse takes battery levels from 0% through 80% as inputs, and $B^{-1}(32) = 6$.

CHALLENGE

$f^{-1}(x) = (4x + 3)/(2 - x)$. The input -4 is excluded from f, and the input 2 is excluded from f^{-1}. The inverse's domain excludes 2 because f can never output 2; f's excluded input -4 is, in turn, excluded from the inverse's range.

Accept equivalent forms of inverse equations and any two correct points that determine a graphed inverse line. For the restricted-domain problems, accept interval notation for domains and ranges.