LESSON MAKER PRO

Powers on Repeat

Find any power of i by spotting a four-step cycle.

HIGH SCHOOL · A2.N.1.1

A2.N.1.1 "Find the value of i^n for any whole number n."

NAME _____ DATE _____

THE FOUR-STEP CYCLE

The powers of i repeat every four steps because $i^4 = 1$. Divide the exponent by 4: a remainder of 0, 1, 2, or 3 gives 1, i, -1, or -i, respectively.

WORKED EXAMPLE

Find i^{27}.

1. $27 = 4 * 6 + 3$.
2. $i^{27} = (i^4)^6 * i^3$.
3. $1^6 * (-i) = -i$.

Answer: -i

PRACTICE 6 problems

1 Find i^0.

ANSWER _____

2 Find i^{14}.

ANSWER _____

3 Find i^{41}.

ANSWER _____

4 Find i^{76}.

ANSWER _____

5 Find i^{99}.

ANSWER _____

6 Find i^{2026}.

ANSWER _____

APPLY IT Show your work

1 In a game, multiplying a position by i rotates it 90° counterclockwise around the origin. A marker starts at 1. After 37 rotations, what complex number gives its position?

WORK SPACE

ANSWER _____

2 An audio app models each quarter-turn phase change by multiplying a signal's complex value by i. A signal starts at -1 and undergoes 11 changes. What is its final value?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Find the smallest whole number n greater than 200 for which $i^n = -i$. Explain how you know it is the smallest.

WORK SPACE AND ANSWER

ANSWER KEY

Powers on Repeat

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find i^0.	**1**
2	Find i^{14}.	**-1**
3	Find i^{41}.	**i**
4	Find i^{76}.	**1**
5	Find i^{99}.	**-i**
6	Find i^{2026}.	**-1**

PART 2 · APPLY IT

1. **i** The position is i^{37}. Since 37 has a remainder of 1 when divided by 4, $i^{37} = i$.
2. **i** The final value is $-1 * i^{11}$. Since $i^{11} = -i$, the result is i.

CHALLENGE

n = 203. It is the first whole number greater than 200 with a remainder of 3 when divided by 4.

Accept equivalent expressions that simplify to the stated complex value. For the challenge, the explanation must establish why 201 and 202 do not work.