LESSON MAKER PRO

Logic Under Test

Use conditions, counterexamples, and algebra to test conclusions.

HIGH SCHOOL · G.RL.1.2

G.RL.1.2 "Analyze and draw conclusions based on a set of conditions using inductive and deductive reasoning. Recognize the logical relationships between a conditional statement and its inverse, converse, and contrapositive."

NAME _____ DATE _____

FROM CONDITIONS TO CONCLUSIONS

For a statement in the form "if P, then Q," the contrapositive, "if not Q, then not P," always has the same truth value. The converse, "if Q, then P," and inverse, "if not P, then not Q," may be false. A few examples can suggest a pattern, but they do not prove it for every number.

WORKED EXAMPLE

If x is a multiple of 22, then x is a multiple of 11. Write the contrapositive and explain why it is true.

1. P means "x is a multiple of 22." Q means "x is a multiple of 11."
2. The contrapositive has the form "if not Q, then not P."
3. Since 22 = 2 × 11, every multiple of 22 is a multiple of 11.

Answer: If x is not a multiple of 11, then x is not a multiple of 22. It is true.

PRACTICE 6 problems

1 If x > 4, then x > 1. Write the converse. Is it true? If not, give a counterexample.

ANSWER _____

2 If x < -3, then x < 2. Write the contrapositive. Is it true?

ANSWER _____

3 If x = -5, then x < 0. Write the inverse. Is it true? If not, give a counterexample.

ANSWER _____

4 If x < 6, then 2x < 12. Is "If 2x >= 12, then x >= 6" the converse, inverse, or contrapositive? Is it true?

ANSWER _____

5 If x < 0, then x < 8. Given x >= 8, what must be true about x? Name the logical form you used.

ANSWER _____

6 A student tests x = 1, 2, and 3 and finds that $x^2 + x$ is even. Do these tests prove the claim for every integer x? Give an algebraic reason the claim is true.

ANSWER _____

APPLY IT Show your work

1 A club sells event wristbands for $12 each and charges a one-time $4 order fee. If an order has at least 3 wristbands, its total is at least $40. Maya's total was $28. Use the contrapositive to find what you can conclude about her order, then find how many wristbands she bought.

WORK SPACE

ANSWER _____

2 In a gaming tournament, each win earns 2 points and each draw earns 1 point. If a player has 4 wins and no draws, the player earns 8 points. Jordan earned 8 points. Must Jordan have had 4 wins and no draws? Give a different combination that earns 8 points and name the related statement that fails.

WORK SPACE

ANSWER _____

CHALLENGE One step up

For real x, consider: If $x^2 = 16$, then $x = 4$. Write the converse and contrapositive. Decide whether each of these three statements is true. Give a counterexample for every false statement.

WORK SPACE AND ANSWER

ANSWER KEY

Logic Under Test

HIGH SCHOOL · G.RL.1.2

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	If x > 4, then x > 1. Write the converse. Is it true? If not, give a counterexample.	**Converse: If x > 1, then x > 4. False; x = 2 is a counterexample.**
2	If x < -3, then x < 2. Write the contrapositive. Is it true?	**If x >= 2, then x >= -3. True.**
3	If x = -5, then x < 0. Write the inverse. Is it true? If not, give a counterexample.	**Inverse: If x != -5, then x >= 0. False; x = -2 is a counterexample.**
4	If x < 6, then 2x < 12. Is "If 2x >= 12, then x >= 6" the converse, inverse, or contrapositive? Is it true?	**Contrapositive. True, because dividing 2x >= 12 by 2 gives x >= 6.**
5	If x < 0, then x < 8. Given x >= 8, what must be true about x? Name the logical form you used.	**x >= 0. This follows from the contrapositive.**
6	A student tests x = 1, 2, and 3 and finds that $x^2 + x$ is even. Do these tests prove the claim for every integer x? Give an algebraic reason the claim is true.	**No, three tests do not prove a claim for every integer. Since $x^2 + x = x(x + 1)$, it is the product of consecutive integers, one of which is even.**

PART 2 · APPLY IT

1. **Maya bought fewer than 3 wristbands. She bought 2.** Since $28 < $40, the contrapositive shows that Maya bought fewer than 3 wristbands. Solving 12x + 4 = 28 gives x = 2.
2. **No. Jordan could have had 3 wins and 2 draws. The converse fails.** Three wins and two draws earn 3(2) + 2(1) = 8 points. Earning 8 points does not require 4 wins and no draws.

CHALLENGE

Original: false; x = -4 is a counterexample. Converse: If x = 4, then $x^2 = 16$. True. Contrapositive: If x != 4, then x^2 != 16. False; x = -4 is a counterexample.

Accept equivalent wording for negated inequalities. For problem 6, require a general argument about consecutive integers rather than additional examples.