LESSON MAKER PRO

Prove or Disprove

Test claims, find counterexamples, and check reasoning.

HIGH SCHOOL · G.RL.1.3

G.RL.1.3 "Assess the validity of a logical argument and give counterexamples to disprove a statement."

NAME _____ DATE _____

TEST THE CLAIM

A claim about every value is false if you find one value that breaks it. Substitute your value and compare the results. One value that works does not prove a claim about every value.

WORKED EXAMPLE

A student claims that $x + 23 > 40$ for every positive integer x. Test $x = 7$. Does it disprove the claim?

1. Substitute 7 for x: $7 + 23 > 40$.
2. Calculate: $30 > 40$.
3. This comparison is false, so 7 is a counterexample.

Answer: Yes. The claim is false because 30 is not greater than 40.

PRACTICE 6 problems

1 A student claims $x + 4 > 2x$ for every positive integer x. Find a counterexample and show the comparison.

ANSWER _____

2 A student tests $x = 3$, finds that $x^2 >= x$, and says this proves the inequality for every integer x. Does the student's test prove the claim? Explain.

ANSWER _____

3 A student argues that $6x$ is even for every integer x because $6x = 2(3x)$, and $3x$ is an integer. Is the argument valid? Explain.

ANSWER _____

4 A student claims $(x + 2)^2 = x^2 + 4$ for every integer x. Find a counterexample and show both sides.

ANSWER _____

5 A student claims $3x + 1 < 10$ for every positive integer x. Find a counterexample and show the comparison.

ANSWER _____

6 A student claims every integer divisible by 4 is also divisible by 8. Find a counterexample and show both divisions.

ANSWER _____

APPLY IT Show your work

1 A streaming service charges $6 each month plus $3 per movie rental. A student claims that renting more than 4 movies always makes the monthly bill greater than $22. Find a number of rentals that disproves the claim and calculate the bill.

WORK SPACE

ANSWER _____

2 A club sells tickets for $5 each and must pay a $21 room fee. A member claims that selling at least 4 tickets always brings in enough money to pay the fee. Find a number of tickets that disproves the claim and compare the revenue with the fee.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A student claims $x^2 + x$ is odd for every odd integer x because odd times odd is odd. Find a counterexample. Then factor $x^2 + x$ and explain what is true about it for every integer x.

WORK SPACE AND ANSWER

ANSWER KEY

Prove or Disprove

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A student claims $x + 4 > 2x$ for every positive integer x. Find a counterexample and show the comparison.	**x = 5: 5 + 4 = 9 and 2(5) = 10. Since 9 is not greater than 10, the claim is false.**
2	A student tests x = 3, finds that $x^2 >= x$, and says this proves the inequality for every integer x. Does the student's test prove the claim? Explain.	**No. The test shows the inequality works for 3, but one working value does not prove it for every integer.**
3	A student argues that 6x is even for every integer x because 6x = 2(3x), and 3x is an integer. Is the argument valid? Explain.	**Yes. For every integer x, 3x is an integer, so 6x is 2 times an integer.**
4	A student claims $(x + 2)^2 = x^2 + 4$ for every integer x. Find a counterexample and show both sides.	**x = 1: $(1 + 2)^2 = 9$, but $1^2 + 4 = 5$. The claim is false.**
5	A student claims $3x + 1 < 10$ for every positive integer x. Find a counterexample and show the comparison.	**x = 3: 3(3) + 1 = 10. Since 10 is not less than 10, the claim is false.**
6	A student claims every integer divisible by 4 is also divisible by 8. Find a counterexample and show both divisions.	**12 is a counterexample. 12/4 = 3, an integer, but 12/8 = 3/2, not an integer.**

PART 2 · APPLY IT

1. **5 rentals give a bill of $21, which is not greater than $22.** Five is more than four. Calculate 6 + 3(5) = 21, then compare $21 with $22.
2. **Selling 4 tickets brings in $20, which is $1 less than the $21 fee.** Four tickets meet the condition of at least four. Calculate 4(5) = 20 and compare $20 with $21.

CHALLENGE

x = 1 is a counterexample because $1^2 + 1 = 2$, which is even. Since $x^2 + x = x(x + 1)$, it is even for every integer x: of two consecutive integers, one is even.

Accept any counterexample that meets the stated conditions and shows the claim fails. In problem 2, the claim is true, but the student's single test does not prove it.