LESSON MAKER PRO

Undo the Function

Find inverses and choose domains that make them possible.

HIGH SCHOOL · PC.F.1.5

PC.F.1.5 "Determine if a function has an inverse. Algebraically and graphically find the inverse or define any restrictions on the domain that meet the requirement for invertibility, and find the inverse on the restricted domain."

NAME _____ DATE _____

WHEN CAN YOU UNDO IT?

A function has an inverse function when each output comes from only one input. On a graph, no horizontal line can cross the function more than once. To find an inverse algebraically, write y=f(x), switch x and y, then solve for y. If two inputs give the same output, you can sometimes restrict the domain to one side of a turning point.

WORKED EXAMPLE

Find the inverse of f(x)=9x+17.

1. Write y=9x+17.
2. Switch x and y: x=9y+17.
3. Subtract 17: x-17=9y.
4. Divide by 9: y=(x-17)/9.

Answer: $f^{-1}(x)=(x-17)/9$

PRACTICE 6 problems

1 Find the inverse of f(x)=2x-3.

ANSWER _____

2 Find the inverse of f(x)=(x+4)/3.

ANSWER _____

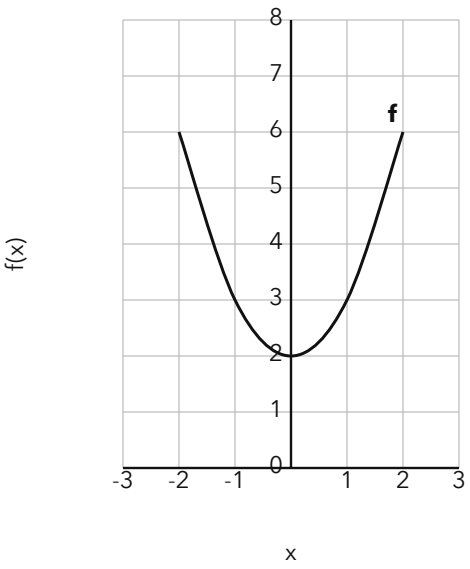
3 The function $f(x)=x^2-4$ is not invertible on all real numbers. Restrict its domain to x>=0, then find its inverse.

ANSWER _____

4 Find the inverse of $f(x)=(x-1)^2+2$ when its domain is x<=1. State the inverse's domain.

ANSWER _____

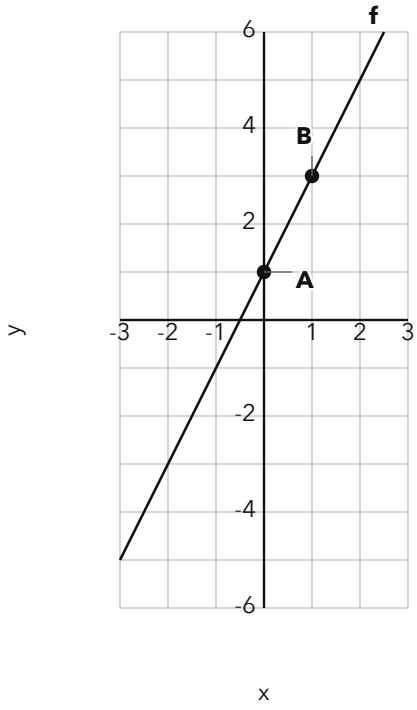
5 The graph shows $f(x)=x^2+2$. Use the graph to explain why f has no inverse function on all real numbers. Give one domain restriction that makes it invertible.



ANSWER _____

_____

6 Is the graphed function one-to-one? Use its line and marked points to find $f^{-1}(x)$.



ANSWER _____

_____

APPLY IT Show your work

1 A ride-share trip costs P(d)=4+2d dollars for d miles, where d>=0. Find the inverse function and its domain. Then use it to find the distance of an $18 trip.

WORK SPACE

ANSWER _____

2 In a video game, a character's height is $h(t)=4-(t-2)^2$ meters for 0<=t<=4 seconds. The same height can occur on the way up and the way down. Restrict the time domain to the descent, find the inverse time function t(h), and find when the character reaches 3 meters on the way down.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Consider $f(x)=x^2-6x+10$. Explain why it has no inverse function on all real numbers. Then restrict its domain to x<=3 and find the inverse, including its domain.

WORK SPACE AND ANSWER

Undo the Function

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the inverse of f(x)=2x-3.	**$f^{-1}(x)=(x+3)/2$**
2	Find the inverse of f(x)=(x+4)/3.	**$f^{-1}(x)=3x-4$**
3	The function $f(x)=x^2-4$ is not invertible on all real numbers. Restrict its domain to x>=0, then find its inverse.	**On the restricted domain x>=0, $f^{-1}(x)=\text{sqrt}(x+4)$, with inverse domain x>=-4.**
4	Find the inverse of $f(x)=(x-1)^2+2$ when its domain is x<=1. State the inverse's domain.	**$f^{-1}(x)=1-\text{sqrt}(x-2)$, with domain x>=2.**
5	The graph shows $f(x)=x^2+2$. Use the graph to explain why f has no inverse function on all real numbers. Give one domain restriction that makes it invertible.	**The graph has two inputs with the same output, such as f(-1)=f(1)=3. Restrict the domain to x>=0 or x<=0.**
6	Is the graphed function one-to-one? Use its line and marked points to find $f^{-1}(x)$.	**Yes. The line is f(x)=2x+1, so $f^{-1}(x)=(x-1)/2$.**

PART 2 · APPLY IT

1. **$P^{-1}(x)=(x-4)/2$ for x>=4. An $18 trip is 7 miles.** Switch the cost and distance variables, then solve for distance. Substitute 18 into the inverse.
2. **Restrict time to 2<=t<=4. Then $t(h)=2+\text{sqrt}(4-h)$ for 0<=h<=4, and the character reaches 3 meters at t=3 seconds.** On the descent, t-2 is nonnegative, so solve $h=4-(t-2)^2$ using the positive square root. Substitute h=3.

CHALLENGE

$f(x)=(x-3)^2+1$, so inputs equally far from 3 give the same output, such as f(2)=f(4)=2. With x<=3, $f^{-1}(x)=3-\text{sqrt}(x-1)$, and the inverse's domain is x>=1.

Accept equivalent algebraic forms. For practice problem 5, either restriction x>=0 or x<=0 is valid if the explanation identifies repeated outputs.