LESSON MAKER PRO

Reverse the Rule

Find and interpret inverse functions

HIGH SCHOOL · PC.F.1

PC.F.1 "Analyze functions and relations. Interpret characteristics of a function defined by an expression in the context of the situation. Sketch the graph of a function that models a relationship between two quantities, identifying key features..."

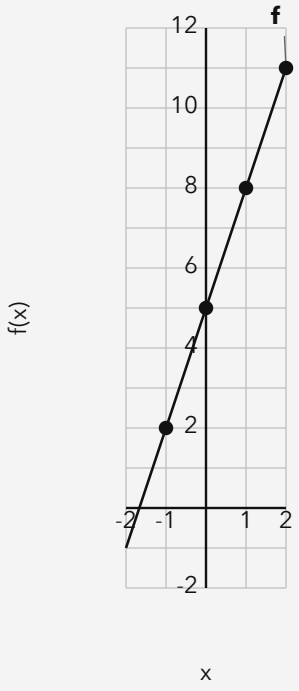
NAME _____ DATE _____

THE BIG IDEA

An inverse swaps inputs and outputs: a point (a, b) on a graph becomes (b, a) on its inverse. An inverse is a function only if each output of the original function comes from exactly one input. Sometimes you can restrict the domain to make that possible.

WORKED EXAMPLE

Use the graph to find $f^{-1}(8)$.



1. Find the point on the graph with output 8: (1, 8).
2. Swap the coordinates to get the inverse point: (8, 1).

Answer: $f^{-1}(8) = 1$

PRACTICE 6 problems

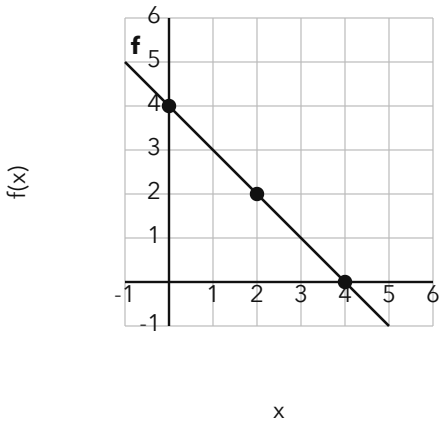
1 Use the table to find $f^{-1}(-2)$.

VALUES OF F

x	f(x)
-3	-6
-1	-2
0	0
2	4

ANSWER _____

2 Use the graph to find $f^{-1}(0)$.



ANSWER _____

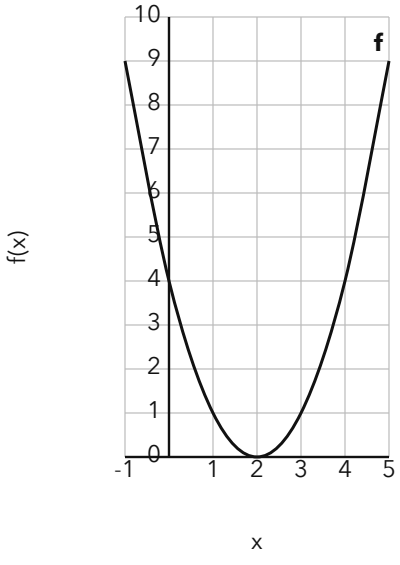
3 Find the inverse of $f(x) = 4x - 3$.

ANSWER _____

4 Find the inverse of $f(x) = x^2$ for $x >= 0$.

ANSWER _____

5 Does the graphed function have an inverse that is a function on its full domain? Give two inputs with the same output to explain your answer.



ANSWER _____

6 For $f(x) = (x + 1)^2$ with $x <= -1$, find $f^{-1}(x)$ and state its domain.

ANSWER _____

APPLY IT Show your work

1 A streaming plan costs $12 each month plus $3 per movie rented. Let C(m) = 12 + 3m, where m is the number of movies. Find $C^{-1}(c)$. Then explain what $C^{-1}(27)$ means.

WORK SPACE

ANSWER _____

2 A model for a scooter's braking distance is $d(v) = v^2/4$, where v >= 0 is speed in feet per second and d is distance in feet. Find $d^{-1}(s)$. What speed does the model give for a braking distance of 9 feet?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Let $f(x) = x^2 - 6x + 11$. Restrict its domain to x >= 3. Find $f^{-1}(x)$ and its domain. Explain why the inverse would not be a function without the restriction.

WORK SPACE AND ANSWER

Reverse the Rule

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Use the table to find $f^{-1}(-2)$.	**$f^{-1}(-2) = -1$**
2	Use the graph to find $f^{-1}(0)$.	**$f^{-1}(0) = 4$**
3	Find the inverse of $f(x) = 4x - 3$.	**$f^{-1}(x) = (x + 3)/4$**
4	Find the inverse of $f(x) = x^2$ for $x >= 0$.	**$f^{-1}(x) = \text{sqrt}(x)$**
5	Does the graphed function have an inverse that is a function on its full domain? Give two inputs with the same output to explain your answer.	**No. Inputs 1 and 3 both have output 1.**
6	For $f(x) = (x + 1)^2$ with $x <= -1$, find $f^{-1}(x)$ and state its domain.	**$f^{-1}(x) = -1 - \text{sqrt}(x)$; domain: $x >= 0$**

PART 2 · APPLY IT

1. **$C^{-1}(c) = (c - 12)/3$. A $27 monthly bill means 5 movies were rented.** Solve $c = 12 + 3m$ for m. Substitute 27 for c to get $m = 5$.
2. **$d^{-1}(s) = 2\text{sqrt}(s)$. The speed is 6 feet per second.** Solve $s = v^2/4$ for the nonnegative speed v. At $s = 9$, $v = 2\text{sqrt}(9) = 6$.

CHALLENGE

Since $f(x) = (x - 3)^2 + 2$ and $x >= 3$, $f^{-1}(x) = 3 + \text{sqrt}(x - 2)$, with domain $x >= 2$. Without the restriction, $f(2) = f(4) = 3$, so the inverse would assign two outputs to 3.

Accept equivalent inverse expressions and any valid pair of distinct inputs with the same output for the graph question. For the challenge, accept any correct example of repeated outputs on the unrestricted domain.