LESSON MAKER PRO

Functions in Sequence

Build composite functions and check where they work.

HIGH SCHOOL · PC.F.2.1

PC.F.2.1 "Model relationships through composition, and attend to the restrictions of the domain."

NAME _____ DATE _____

THE BIG IDEA

In f(g(x)), use the output of g as the input of f. To find the domain, keep only inputs that work in g and make its output a valid input for f. Check those restrictions even if the final rule simplifies.

WORKED EXAMPLE

Let f(x) = sqrt(x - 41) and g(x) = 7x + 19. Find f(g(x)) and its domain.

1. Substitute g(x) into f: f(g(x)) = sqrt((7x + 19) - 41).
2. Simplify: f(g(x)) = sqrt(7x - 22).
3. Require 7x - 22 >= 0, so x >= 22/7.

Answer: f(g(x)) = sqrt(7x - 22); domain: x >= 22/7.

PRACTICE 6 problems

1 Let f(x) = sqrt(x + 1) and g(x) = x - 5. Find f(g(x)) and its domain.

ANSWER _____

2 Let f(x) = 1/(x - 3) and g(x) = x + 2. Find f(g(x)) and its domain.

ANSWER _____

3 Let f(x) = x^2 + 1 and g(x) = sqrt(x - 9). Find f(g(x)) and its domain.

ANSWER _____

4 Let f(x) = sqrt(x) and g(x) = 1/(x + 4). Find f(g(x)) and its domain.

ANSWER _____

5 Let f(x) = 1/(x + 1) and g(x) = x^2 - 4. Find f(g(x)) and its domain.

ANSWER _____

6 Let f(x) = sqrt(x - 1) and g(x) = x^2 - 9. Find f(g(x)) and its domain.

ANSWER _____

APPLY IT Show your work

1 You are making a square image for a club poster. The poster has x cm^2 of space, and the title uses 18 cm^2. Let g(x) = x - 18 be the area left for the image, and let f(A) = sqrt(A) give the side length of a square with area A. Find f(g(x)), state its domain, and find the image's side length when x = 43.

WORK SPACE

ANSWER _____

2 Your group splits a $36 rideshare fare, but one person paid for snacks and does not pay for the ride. Let x be the total number of people, g(x) = x - 1 be the number who pay, and f(n) = 36/n be each paying person's share in dollars. Find f(g(x)), state the valid values of x, and find each share when x = 5.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Let f(x) = 1/(x - 2), g(x) = x^2 - 1, and h(x) = sqrt(x + 4). Find f(g(h(x))) and its domain. Explain why the simplified rule alone does not give the full domain.

WORK SPACE AND ANSWER

ANSWER KEY

Functions in Sequence

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Let f(x) = sqrt(x + 1) and g(x) = x - 5. Find f(g(x)) and its domain.	**f(g(x)) = sqrt(x - 4); domain: x >= 4.**
2	Let f(x) = 1/(x - 3) and g(x) = x + 2. Find f(g(x)) and its domain.	**f(g(x)) = 1/(x - 1); domain: x != 1.**
3	Let f(x) = x^2 + 1 and g(x) = sqrt(x - 9). Find f(g(x)) and its domain.	**f(g(x)) = x - 8; domain: x >= 9.**
4	Let f(x) = sqrt(x) and g(x) = 1/(x + 4). Find f(g(x)) and its domain.	**f(g(x)) = sqrt(1/(x + 4)); domain: x > -4.**
5	Let f(x) = 1/(x + 1) and g(x) = x^2 - 4. Find f(g(x)) and its domain.	**f(g(x)) = 1/(x^2 - 3); domain: x != -sqrt(3) and x != sqrt(3).**
6	Let f(x) = sqrt(x - 1) and g(x) = x^2 - 9. Find f(g(x)) and its domain.	**f(g(x)) = sqrt(x^2 - 10); domain: x <= -sqrt(10) or x >= sqrt(10).**

PART 2 · APPLY IT

1. **f(g(x)) = sqrt(x - 18) cm; domain: x >= 18; at x = 43, the side length is 5 cm.** The area left for the image must be nonnegative, so x - 18 >= 0. At x = 43, the remaining area is 25 cm^2, giving a side length of 5 cm.
2. **f(g(x)) = 36/(x - 1) dollars; x is a whole number at least 2; when x = 5, each paying person owes $9.** There must be at least one paying person, so x is a whole number at least 2. With 5 people, 4 pay, and 36/4 = 9.

CHALLENGE

f(g(h(x))) = 1/(x + 1). Its domain is x >= -4 and x != -1. The square root in h(x) requires x >= -4, even though that restriction is not visible in the simplified rule.

Accept equivalent composite expressions and equivalent domain notation. In problem 3 and the challenge, the domain of the original composition still applies after simplification.