LESSON MAKER PRO

Inverse Detectives

Check whether two functions undo each other.

HIGH SCHOOL · PC.F.2.4

PC.F.2.4 "Verify by analytical methods that one function is the inverse of another."

NAME _____ DATE _____

THE TWO-WAY TEST

To verify that two functions are inverses, find both f(g(x)) and g(f(x)). Each composition must equal x for every input allowed by its domain. One composition equaling x is not enough.

WORKED EXAMPLE

Are f(x) = 7x - 11 and g(x) = (x + 11)/7 inverses?

1. f(g(x)) = 7((x + 11)/7) - 11
2. f(g(x)) = x + 11 - 11 = x
3. g(f(x)) = ((7x - 11) + 11)/7
4. g(f(x)) = 7x/7 = x

Answer: Yes. Both compositions equal x for all real x.

PRACTICE 6 problems

1 For f(x) = 3x + 5 and g(x) = (x - 5)/3, find both compositions. Are the functions inverses?

ANSWER _____

2 For f(x) = 4x - 2 and g(x) = (x - 2)/4, find both compositions. Are the functions inverses?

ANSWER _____

3 For f(x) = x/2 + 6 and g(x) = 2x - 12, find both compositions. Are the functions inverses?

ANSWER _____

4 Let f(x) = x^2 + 4 for x >= 0, and let g(x) = sqrt(x - 4) for x >= 4. Find both compositions on their allowed domains. Are the functions inverses?

ANSWER _____

5 Let f(x) = 5/x and g(x) = 5/x, each defined for x != 0. Find both compositions. Are the functions inverses?

ANSWER _____

6 For f(x) = x^3 - 3 and g(x) = cube root of (x + 3), find both compositions. Are the functions inverses?

ANSWER _____

APPLY IT Show your work

1 At a festival, a wristband costs $8 plus $3 for each ride. The function f(x) = 3x + 8 gives the cost for x rides, and g(x) = (x - 8)/3 gives the number of rides for a possible total cost x. Verify both compositions. What does g(f(4)) mean?

WORK SPACE

ANSWER _____

2 A square profile picture has x pixels on each side. Its pixel count is $f(x) = x^2$. A designer uses g(x) = sqrt(x) to find the side length from a possible square pixel count x. Verify both compositions. What does g(f(6)) mean?

WORK SPACE

ANSWER _____

CHALLENGE One step up

A student claims that $f(x) = x^2 + 9$ for all real x and g(x) = sqrt(x - 9) for x >= 9 are inverses because f(g(x)) = x. Find g(f(x)), explain why the claim fails, and give a domain restriction for f that makes the functions inverses.

WORK SPACE AND ANSWER

Inverse Detectives

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For f(x) = 3x + 5 and g(x) = (x - 5)/3, find both compositions. Are the functions inverses?	**f(g(x)) = x and g(f(x)) = x. Yes.**
2	For f(x) = 4x - 2 and g(x) = (x - 2)/4, find both compositions. Are the functions inverses?	**f(g(x)) = x - 4 and g(f(x)) = x - 1. No.**
3	For f(x) = x/2 + 6 and g(x) = 2x - 12, find both compositions. Are the functions inverses?	**f(g(x)) = x and g(f(x)) = x. Yes.**
4	Let f(x) = x^2 + 4 for x >= 0, and let g(x) = sqrt(x - 4) for x >= 4. Find both compositions on their allowed domains. Are the functions inverses?	**f(g(x)) = x for x >= 4, and g(f(x)) = x for x >= 0. Yes.**
5	Let f(x) = 5/x and g(x) = 5/x, each defined for x != 0. Find both compositions. Are the functions inverses?	**f(g(x)) = x and g(f(x)) = x for x != 0. Yes.**
6	For f(x) = x^3 - 3 and g(x) = cube root of (x + 3), find both compositions. Are the functions inverses?	**f(g(x)) = x and g(f(x)) = x for all real x. Yes.**

PART 2 · APPLY IT

1. **f(g(x)) = x for possible total costs, and g(f(x)) = x for nonnegative whole numbers of rides. The value g(f(4)) = 4 means that the cost of four rides corresponds to four rides.** Substitute each function into the other and simplify. Because f(4) = 20 and g(20) = 4, g(f(4)) returns the original ride count.
2. **f(g(x)) = x for possible square pixel counts, and g(f(x)) = x for nonnegative whole-number side lengths. The value g(f(6)) = 6 means that a picture with 6 pixels on each side has a side length of 6 pixels when its pixel count is converted back.** Squaring sqrt(x) returns x for a possible square pixel count. Also, sqrt(x^2) = x for a nonnegative side length, so g(f(6)) = sqrt(36) = 6.

CHALLENGE

g(f(x)) = sqrt(x^2) = |x|, which is not x when x < 0. For example, g(f(-2)) = 2. Restrict f to x >= 0; with g defined for x >= 9, both compositions then equal x on their allowed domains.

Accept equivalent simplified forms. For restricted-domain items, a correct inverse claim must account for the stated domains.