LESSON MAKER PRO

Function Chain Reactions

Compose functions and check which inputs work.

HIGH SCHOOL · PC.F.2

PC.F.2 "Build functions to model and validate relationships among functions. Model relationships through composition, and attend to the restrictions of the domain. Rewrite a function as a composition of functions..."

NAME _____ **DATE** _____

ORDER AND DOMAIN

In f(g(x)), apply g first, then use its output as the input to f. An input belongs to the domain of the composition only if both steps are defined. Simplifying an expression does not restore an input that was undefined in either step.

WORKED EXAMPLE

Let f(x) = sqrt(x - 23) and g(x) = 5x + 23. Find f(g(x)), its domain, and f(g(20)).

1. Substitute g(x) into f: f(g(x)) = sqrt((5x + 23) - 23).
2. Simplify: f(g(x)) = sqrt(5x).
3. Require 5x >= 0, so x >= 0.
4. At x = 20, f(g(20)) = sqrt(100) = 10.

Answer: f(g(x)) = sqrt(5x); domain: x >= 0; f(g(20)) = 10.

PRACTICE 6 problems

1 Let f(x) = 2x + 1 and g(x) = x^2 - 4. Find f(g(x)) and f(g(3)).

ANSWER _____

2 Let f(x) = sqrt(x - 4) and g(x) = 2x + 1. Find f(g(x)) and its domain.

ANSWER _____

3 Let f(x) = 1/(x - 2) and g(x) = x + 4. Find f(g(x)) and its domain.

ANSWER _____

4 Let f(x) = x^2 + 2 and g(x) = sqrt(x - 1). Find f(g(x)) and its domain.

ANSWER _____

5 Let h(x) = sqrt(4x + 1). Write h as f(g(x)) using f(x) = sqrt(x) and g(x) = 4x + 1. State the domain of h.

ANSWER _____

6 Let f(x) = 1/(x - 1) and g(x) = x^2 - 8. Find f(g(x)) and its domain.

ANSWER _____

APPLY IT Show your work

1 A concert ticket has a listed price of x dollars. A student discount changes the price to g(x) = 3x/4. The ticket site then adds a service fee using f(x) = x + 2. Write the final price as a function of x. What is the final price when the listed price is $24?

WORK SPACE

ANSWER _____

2 An online store takes $4 off an order, then adds 10% sales tax. For an original price of x dollars, where x >= 4, the coupon is c(x) = x - 4 and the tax is t(x) = 11x/10. Write the final price as a composition and find it for a $30 order.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Let h(x) = sqrt(2x - 4) with domain x >= 2, and let k(x) = (x^2 + 4)/2 with domain x >= 0. Show that h and k are inverses by finding both h(k(x)) and k(h(x)). State the domain on which each composition equals x.

WORK SPACE AND ANSWER

ANSWER KEY

Function Chain Reactions

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Let f(x) = 2x + 1 and g(x) = x^2 - 4. Find f(g(x)) and f(g(3)).	**f(g(x)) = $2x^2$ - 7; f(g(3)) = 11.**
2	Let f(x) = sqrt(x - 4) and g(x) = 2x + 1. Find f(g(x)) and its domain.	**f(g(x)) = sqrt(2x - 3); domain: x >= 3/2.**
3	Let f(x) = 1/(x - 2) and g(x) = x + 4. Find f(g(x)) and its domain.	**f(g(x)) = 1/(x + 2); domain: x != -2.**
4	Let f(x) = x^2 + 2 and g(x) = sqrt(x - 1). Find f(g(x)) and its domain.	**f(g(x)) = x + 1; domain: x >= 1.**
5	Let h(x) = sqrt(4x + 1). Write h as f(g(x)) using f(x) = sqrt(x) and g(x) = 4x + 1. State the domain of h.	**h(x) = f(g(x)); domain: x >= -1/4.**
6	Let f(x) = 1/(x - 1) and g(x) = x^2 - 8. Find f(g(x)) and its domain.	**f(g(x)) = 1/(x^2 - 9); domain: x != -3 and x != 3.**

PART 2 · APPLY IT

1. **f(g(x)) = 3x/4 + 2; the final price is $20.** Apply the discount before adding the fee. At x = 24, the discounted price is $18, then the fee brings it to $20.
2. **t(c(x)) = 11(x - 4)/10; the final price is $28.60.** Apply the coupon before the tax. The $30 order becomes $26, and 11(26)/10 = $28.60.

CHALLENGE

h(k(x)) = sqrt(x^2) = x for x >= 0. k(h(x)) = ((2x - 4) + 4)/2 = x for x >= 2. Both compositions equal x on their stated domains, so h and k are inverses.

Accept equivalent simplified expressions. For the challenge, sqrt(x^2) = x is valid because the domain of k is x >= 0.