LESSON MAKER PRO

# Trig Expression Makeovers

*Use structure and identities to reveal simpler forms.*

**HIGH SCHOOL · PC.T.3.1**

**PC.T.3.1** "Algebraically manipulate the structure of a trigonometric expression to identify ways to rewrite it."

NAME _____ DATE _____

## LOOK FOR A REWRITE

Look for a common factor or a trigonometric identity that makes an expression simpler. You can replace tan x with sin x/cos x or use $\sin^2 x + \cos^2 x = 1$. A rewritten expression is equivalent wherever the original expression is defined.

## WORKED EXAMPLE

Rewrite $7 \tan x \cos x + 7 \sin x$ as a single sine term.

1. Replace tan x with sin x/cos x.
2. $7(\sin x/\cos x)\cos x + 7 \sin x = 7 \sin x + 7 \sin x$
3. Combine like terms: $7 \sin x + 7 \sin x = 14 \sin x$.

**Answer: 14 sin x**

## PRACTICE 6 problems

**1** Rewrite $(1 - \sin^2 x)/\cos x$ without a fraction.

**ANSWER** _____

**2** Rewrite $(\sec^2 x - 1)/\tan x$ as a single trigonometric function.

**ANSWER** _____

**3** Rewrite $\sin^2 x/(1 + \cos x)$ without a fraction.

**ANSWER** _____

**4** Factor, then simplify $(\sin x + \sin x \cos x)/(1 + \cos x)$.

**ANSWER** _____

**5** Simplify $(3 \sin x \cos x)/(6 \cos x)$.

**ANSWER** _____

**6** Rewrite $1/(1 - \sin x) - 1/(1 + \sin x)$ as one fraction in sin x and cos x.

**ANSWER** _____

## APPLY IT Show your work

**1** A music producer models a blended sound wave by $A(x) = 4 \sin x \cos^2 x + 4 \sin^3 x$, where x is a phase angle. Rewrite A(x) as a single sine term.

WORK SPACE

**ANSWER** _____

**2** A game developer tests two formulas for a camera's turn rate: $R_1(x) = 2 \sin x/(1 + \cos x)$ and $R_2(x) = 2(1 - \cos x)/\sin x$. Show algebraically whether they give the same rate wherever both are defined.

WORK SPACE

**ANSWER** _____

## CHALLENGE One step up

Rewrite $\sin x/(1 + \cos x) + (1 + \cos x)/\sin x$ as a single trigonometric ratio. Show where you use $\sin^2 x + \cos^2 x = 1$.

WORK SPACE AND ANSWER

**ANSWER KEY**

# Trig Expression Makeovers



## PART 1 · PRACTICE

| # | PROBLEM | ANSWER |
|---|---|---|
| 1 | Rewrite $(1 - \sin^2 x)/\cos x$ without a fraction. | **cos x** |
| 2 | Rewrite $(\sec^2 x - 1)/\tan x$ as a single trigonometric function. | **tan x** |
| 3 | Rewrite $\sin^2 x/(1 + \cos x)$ without a fraction. | **1 - cos x** |
| 4 | Factor, then simplify $(\sin x + \sin x \cos x)/(1 + \cos x)$. | **sin x** |
| 5 | Simplify $(3 \sin x \cos x)/(6 \cos x)$. | **sin x/2** |
| 6 | Rewrite $1/(1 - \sin x) - 1/(1 + \sin x)$ as one fraction in sin x and cos x. | **$2 \sin x/\cos^2 x$** |

## PART 2 · APPLY IT

1. **A(x) = 4 sin x** Factor out 4 sin x. The remaining factor is $\cos^2 x + \sin^2 x = 1$.
2. **Yes. $R_1(x) = R_2(x)$ wherever both are defined.** Multiply $R_1(x)$ by $(1 - \cos x)/(1 - \cos x)$. Its denominator becomes $1 - \cos^2 x = \sin^2 x$, and canceling sin x gives $R_2(x)$.

## CHALLENGE

**$[\sin^2 x + (1 + \cos x)^2]/[\sin x(1 + \cos x)] = [2 + 2 \cos x]/[\sin x(1 + \cos x)] = 2/\sin x$. The numerator becomes $2 + 2 \cos x$ because $\sin^2 x + \cos^2 x = 1$.**

Accept equivalent simplified forms where a problem does not require a particular form. All equivalences apply only where the original expressions are defined.