LESSON MAKER PRO

Unmask the Angle

Rewrite trig expressions to find exact solutions.

HIGH SCHOOL · PC.T.3

PC.T.3 "Verify trigonometric identities and solve equations. Algebraically manipulate the structure of a trigonometric expression to identify ways to rewrite it..."

NAME _____ DATE _____

REWRITE, THEN SOLVE

Use $\sin^2 x + \cos^2 x = 1$ to rewrite an equation with just sine or just cosine. Solve the new equation, then keep only values between -1 and 1 for sine or cosine.

WORKED EXAMPLE

Solve $\sin^2 x + (5/2)\cos x = 2$ for $0 \le x < 2\pi$.

1. Replace $\sin^2 x$ with $1 - \cos^2 x$: $1 - \cos^2 x + (5/2)\cos x = 2$.
2. Rearrange and factor: $(\cos x - 1/2)(\cos x - 2) = 0$.
3. Since $\cos x$ cannot equal 2, use $\cos x = 1/2$.

Answer: $x = \pi/3, 5\pi/3$

PRACTICE 6 problems

1 Solve $\sin^2 x + \cos x = 1$ for $0 \le x < 2\pi$.

ANSWER _____

2 Solve $\cos^2 x + \sin x = 1$ for $0 \le x < 2\pi$.

ANSWER _____

3 Solve $\sin^2 x + (1/2)\cos x = 1/2$ for $0 \le x < 2\pi$.

ANSWER _____

4 Solve $\cos^2 x - (1/2)\sin x = 1/2$ for $0 \le x < 2\pi$.

ANSWER _____

5 Solve $\sin^2 x - \cos^2 x = 0$ for $0 \le x < 2\pi$.

ANSWER _____

6 Rewrite $\sin^2 x + 3 \sin x + \cos^2 x$ to make its maximum clear. State the maximum.

ANSWER _____

APPLY IT Show your work

1 A game uses the meter $M(x) = 2 + \cos^2 x + \sin x$, where x is the phase in radians during one cycle. At which phases $0 \le x < 2\pi$ does the meter read 13/4?

WORK SPACE

ANSWER _____

2 An LED music visualizer uses the brightness setting $B(x) = 1 + \sin^2 x - \cos x$, where x is the phase in radians during one cycle. At which phases $0 \le x < 2\pi$ is the setting 9/4?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Solve $\sin^2 x + \sin x = \cos^2 x + \cos x$ for $0 \le x < 2\pi$. Explain how you know your list is complete.

WORK SPACE AND ANSWER

Unmask the Angle

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Solve $\sin^2 x + \cos x = 1$ for $0 \le x < 2\pi$.	**$x = 0, \pi/2, 3\pi/2$**
2	Solve $\cos^2 x + \sin x = 1$ for $0 \le x < 2\pi$.	**$x = 0, \pi/2, \pi$**
3	Solve $\sin^2 x + (1/2)\cos x = 1/2$ for $0 \le x < 2\pi$.	**$x = 0, 2\pi/3, 4\pi/3$**
4	Solve $\cos^2 x - (1/2)\sin x = 1/2$ for $0 \le x < 2\pi$.	**$x = \pi/6, 5\pi/6, 3\pi/2$**
5	Solve $\sin^2 x - \cos^2 x = 0$ for $0 \le x < 2\pi$.	**$x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$**
6	Rewrite $\sin^2 x + 3 \sin x + \cos^2 x$ to make its maximum clear. State the maximum.	**$1 + 3 \sin x$; maximum: 4**

PART 2 · APPLY IT

1. **$x = \pi/6, 5\pi/6$** Set $2 + \cos^2 x + \sin x = 13/4$ and replace $\cos^2 x$ with $1 - \sin^2 x$. This gives $(\sin x - 1/2)^2 = 0$, so $\sin x = 1/2$.
2. **$x = 2\pi/3, 4\pi/3$** Set $1 + \sin^2 x - \cos x = 9/4$ and replace $\sin^2 x$ with $1 - \cos^2 x$. This gives $(\cos x + 1/2)^2 = 0$, so $\cos x = -1/2$.

CHALLENGE

$x = \pi/4, \pi, 5\pi/4, 3\pi/2$. Rearranging and factoring gives $(\sin x - \cos x)(\sin x + \cos x + 1) = 0$. If $\sin x = \cos x$, then $x = \pi/4$ or $5\pi/4$. If $\sin x + \cos x = -1$, squaring shows $\sin x \cos x = 0$; the negative sum then gives $x = \pi$ or $3\pi/2$. These are all cases from the two factors.

Accept solutions in any order. In problem 6, accept $3 \sin x + 1$ as the equivalent form; its maximum occurs when $\sin x = 1$.