LESSON MAKER PRO

Chance Over Time

Compare expected chances with observed results.

HIGH SCHOOL · S.P.1.2

S.P.1.2 "Describe the relationship between theoretical and empirical probabilities using the Law of Large Numbers."

NAME _____ DATE _____

THE BIG IDEA

Theoretical probability comes from what could happen. Empirical probability is the number of successes divided by the number of trials. The Law of Large Numbers says that, over many independent trials, empirical probability tends toward theoretical probability, but a larger sample is not guaranteed to be closer.

WORKED EXAMPLE

A spinner has 8 equal sections, 3 blue. Use the table to compare the results with the theoretical probability of blue.

SPINNER RESULTS

Spins	Blue results
16	8
80	32

1. Theoretical P(blue) = 3/8.
2. Empirical P(blue) = 8/16 = 1/2 and 32/80 = 2/5.
3. The distances from 3/8 are 1/8 and 1/40.

Answer: The 80-spin result is closer to the theoretical probability.

PRACTICE 4 problems

1 A fair number cube lands on at least 5 in 12 of 30 rolls. Find the theoretical and empirical probabilities.

ANSWER _____

2 A bag holds 7 blue and 7 gold tokens. A token is drawn and replaced 40 times; 18 draws are blue. Find the theoretical and empirical probabilities of blue.

ANSWER _____

3 A fair coin lands heads 9 times in 12 flips and 62 times in 120 flips. Find both empirical probabilities. Which is closer to the theoretical probability?

ANSWER _____

4 A fair coin lands heads 15 times in 25 flips. About how many heads would you expect in 250 flips? Is that count guaranteed?

ANSWER _____

APPLY IT Show your work

1 A music app randomly picks one of two tracks; one is a remix. Use the table. Find the theoretical remix probability and both empirical probabilities. Which sample is closer?

REMIX RESULTS

Picks	Remixes
40	24
400	204

ANSWER _____

2 A game wheel has 10 equal sections, 3 with bonus points. In 200 spins, 54 land on a bonus. Find the theoretical and empirical probabilities. Would 2,000 spins guarantee exactly 600 bonuses?

WORK SPACE

ANSWER _____

CHALLENGE One step up

A fair coin lands heads 27 times in 60 flips and 270 times in the next 600 flips. Compare both empirical probabilities with 1/2. Does this contradict the Law of Large Numbers? Explain.

WORK SPACE AND ANSWER

ANSWER KEY

Chance Over Time

HIGH SCHOOL · S.P.1.2

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A fair number cube lands on at least 5 in 12 of 30 rolls. Find the theoretical and empirical probabilities.	**Theoretical: 1/3. Empirical: 12/30 = 2/5.**
2	A bag holds 7 blue and 7 gold tokens. A token is drawn and replaced 40 times; 18 draws are blue. Find the theoretical and empirical probabilities of blue.	**Theoretical: 1/2. Empirical: 18/40 = 9/20.**
3	A fair coin lands heads 9 times in 12 flips and 62 times in 120 flips. Find both empirical probabilities. Which is closer to the theoretical probability?	**9/12 = 3/4 and 62/120 = 31/60. The 120-flip result is closer to 1/2.**
4	A fair coin lands heads 15 times in 25 flips. About how many heads would you expect in 250 flips? Is that count guaranteed?	**About 125 heads. No, the count is not guaranteed.**

PART 2 · APPLY IT

1. **Theoretical: 1/2. Empirical: 24/40 = 3/5 and 204/400 = 51/100. The 400-pick sample is closer.** Compare each empirical probability with 1/2. Their distances are 1/10 and 1/100.
2. **Theoretical: 3/10. Empirical: 54/200 = 27/100. No, 600 bonuses is not guaranteed.** Use bonus sections divided by all sections, then bonus results divided by all spins. Theoretical probability predicts about 600 bonuses in 2,000 spins, not an exact count.

CHALLENGE

Both empirical probabilities equal 9/20, which is 1/20 below 1/2. No. The law describes a long-run trend, not a guarantee that each larger sample will be closer.

Accept equivalent fractions or decimals. In the challenge, students should note that the larger sample is no closer, without claiming this disproves the long-run trend.