LESSON MAKER PRO

Chances That Change

Find conditional probabilities and test independence.

HIGH SCHOOL · S.P.2

S.P.2 "Determine probabilities, including joint probabilities, conditional probabilities, probabilities of independent events, and probabilities of dependent events. Interpret the results. Understand that two events, A and B..."

NAME _____ DATE _____

FOCUS ON THE GIVEN GROUP

To find P(A given B), look only at outcomes in B. Calculate P(A|B) = P(A and B)/P(B), as long as P(B) is not zero. Events are independent when P(A|B) = P(A), or equivalently, when P(A and B) = P(A) × P(B).

WORKED EXAMPLE

At a game expo, 45 visitors were surveyed. Fifteen played a puzzle game, and 9 played the puzzle game and won a sticker. Find P(sticker | puzzle game).

1. P(sticker and puzzle game) = 9/45.
2. P(puzzle game) = 15/45.
3. P(sticker | puzzle game) = (9/45)/(15/45) = 9/15 = 3/5.

Answer: 3/5 of the puzzle-game players won a sticker.

PRACTICE 6 problems

1 Of 24 students, 8 are on the robotics team and 6 are on the robotics team and play guitar. What is the probability that a randomly selected student is on the robotics team and plays guitar?

ANSWER _____

2 Use the table to find P(after-school job | transit pass).

STUDENT SURVEY

	Transit pass	No transit pass
After-school job	12	8
No after-school job	6	14

ANSWER _____

3 Use the table to find P(brings lunch | rides bus). Are bringing lunch and riding the bus independent? Explain using P(brings lunch).

STUDENT TRAVEL AND LUNCH

	Rides bus	Does not ride bus
Brings lunch	12	8
Does not bring lunch	18	12

ANSWER _____

_____

4 For a randomly selected school day, P(rain) = 2/3, P(late bus) = 1/4, and P(rain and late bus) = 1/6. Are rain and a late bus independent? Show the product you compare.

ANSWER _____

_____

5 A bag has 3 blue tokens and 5 green tokens. You draw two tokens without replacement. What is the probability that both are blue?

ANSWER _____

6 Suppose P(A) = 1/4, P(B) = 3/5, and P(A and B) = 3/20. Find P(A|B). Are A and B independent?

ANSWER _____

_____

APPLY IT Show your work

1 A school concert survey recorded whether students followed the band and bought a ticket. If a randomly selected student follows the band, what is the probability they bought a ticket? Interpret your result.

SCHOOL CONCERT SURVEY

	Bought ticket	Did not buy ticket
Follows band	18	6
Does not follow band	12	24

ANSWER _____

2 A playlist has 12 songs, including 5 by your favorite artist. It plays two songs in order without replaying the first. What is the probability that both songs are by your favorite artist? Explain why the second probability changes.

WORK SPACE

ANSWER _____

CHALLENGE One step up

In a group of 60 students, 20 wear a club shirt and 24 carry a reusable bottle. How many students must do both for the events to be independent? Then find P(club shirt | reusable bottle) and explain how it confirms independence.

WORK SPACE AND ANSWER

Chances That Change

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Of 24 students, 8 are on the robotics team and 6 are on the robotics team and play guitar. What is the probability that a randomly selected student is on the robotics team and plays guitar?	**6/24 = 1/4**
2	Use the table to find P(after-school job | transit pass).	**12/(12 + 6) = 2/3**
3	Use the table to find P(brings lunch | rides bus). Are bringing lunch and riding the bus independent? Explain using P(brings lunch).	**P(brings lunch | rides bus) = 12/30 = 2/5. Yes, they are independent because P(brings lunch) = 20/50 = 2/5.**
4	For a randomly selected school day, P(rain) = 2/3, P(late bus) = 1/4, and P(rain and late bus) = 1/6. Are rain and a late bus independent? Show the product you compare.	**Yes. P(rain) × P(late bus) = 2/3 × 1/4 = 1/6, which equals P(rain and late bus).**
5	A bag has 3 blue tokens and 5 green tokens. You draw two tokens without replacement. What is the probability that both are blue?	**3/8 × 2/7 = 3/28**
6	Suppose P(A) = 1/4, P(B) = 3/5, and P(A and B) = 3/20. Find P(A|B). Are A and B independent?	**P(A|B) = (3/20)/(3/5) = 1/4. Yes, because P(A|B) = P(A).**

PART 2 · APPLY IT

1. **18/(18 + 6) = 3/4. Three out of four surveyed students who follow the band bought a ticket.** Use only the 24 students who follow the band. Of those students, 18 bought a ticket.
2. **5/12 × 4/11 = 5/33. After the first song by that artist plays, only 4 of the 11 remaining songs are by that artist.** Multiply the chance that the first song is by the artist by the chance that the second is by the artist, given the first choice. The first song is not available for the second choice.

CHALLENGE

8 students, because 60 × (20/60) × (24/60) = 8. P(club shirt | reusable bottle) = 8/24 = 1/3, which equals P(club shirt) = 20/60 = 1/3.

Accept equivalent fractions and correct decimals. For independence, accept a comparison of either conditional and overall probabilities or joint and multiplied probabilities.