LESSON MAKER PRO

Worth the Risk

Use expected value to compare choices.

HIGH SCHOOL · S.P.3

S.P.3 "Use probability to make decisions. Analyze decisions and strategies using probability concepts and expected values. Analyze decisions about statistical significance based on reported p-values."

NAME _____ DATE _____

EXPECTED NET GAIN

Expected net gain is the average amount you would gain or lose over many tries. Multiply each possible net gain by its probability, then add the results. When money is your only goal, the choice with the higher expected net gain is better on average.

WORKED EXAMPLE

A card game gives you a 2/5 chance of gaining $11 and a 3/5 chance of losing $6. What is the expected net gain?

1. Winning contributes (2/5)($11) = $22/5.
2. Losing contributes (3/5)(-$6) = -$18/5.
3. Add the contributions: $22/5 - $18/5 = $4/5.

Answer: The expected net gain is $4/5 per game.

PRACTICE 6 problems

1 A game gives you a 1/2 chance of gaining $8 and a 1/2 chance of losing $4. Find the expected net gain.

ANSWER _____

2 A game gives you a 1/4 chance of gaining $12 and a 3/4 chance of losing $4. Find the expected net gain.

ANSWER _____

3 A game gives you a 1/3 chance of gaining $10 and a 2/3 chance of losing $8. Find the expected net gain.

ANSWER _____

4 Choice A pays you $2 for sure. Choice B gives you a 1/2 chance of gaining $8 and a 1/2 chance of losing $2. Find Choice B's expected net gain. Which choice has the higher expected net gain?

ANSWER _____

5 You pay $4 to enter a drawing. You have a 1/4 chance of winning $20 and otherwise win nothing. Find your expected net gain after the entry cost.

ANSWER _____

6 A game gives you a 1/8 chance of gaining $32 and a 7/8 chance of losing $8. Find the expected net gain. If your goal is to gain money on average, should you play?

ANSWER _____

APPLY IT Show your work

1 You can enter an esports tournament for $4. You estimate a 1/4 chance of winning a $32 prize and otherwise win nothing. Instead, you could earn $3 helping at the event. Which choice has the higher expected net gain?

WORK SPACE

ANSWER _____

2 You can buy a hoodie for $18 to resell. You estimate a 1/2 chance of selling it for $30 and a 1/2 chance of selling it for $10. Instead, you could sell your old books for a guaranteed $4 profit. Which choice has the higher expected profit?

WORK SPACE

ANSWER _____

CHALLENGE One step up

A mystery pack costs $10. Its contents can be sold for $40 with probability x or for $8 otherwise. You could earn a guaranteed $2 profit instead. What is the smallest value of x for which the mystery pack has at least as much expected profit? Show how you know.

WORK SPACE AND ANSWER

Worth the Risk

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A game gives you a 1/2 chance of gaining $8 and a 1/2 chance of losing $4. Find the expected net gain.	**$2**
2	A game gives you a 1/4 chance of gaining $12 and a 3/4 chance of losing $4. Find the expected net gain.	**$0**
3	A game gives you a 1/3 chance of gaining $10 and a 2/3 chance of losing $8. Find the expected net gain.	**-$2**
4	Choice A pays you $2 for sure. Choice B gives you a 1/2 chance of gaining $8 and a 1/2 chance of losing $2. Find Choice B's expected net gain. Which choice has the higher expected net gain?	**Choice B has an expected net gain of $3. Choose B.**
5	You pay $4 to enter a drawing. You have a 1/4 chance of winning $20 and otherwise win nothing. Find your expected net gain after the entry cost.	**$1**
6	A game gives you a 1/8 chance of gaining $32 and a 7/8 chance of losing $8. Find the expected net gain. If your goal is to gain money on average, should you play?	**The expected net gain is -$3. Do not play.**

PART 2 · APPLY IT

1. **Enter the tournament. Its expected net gain is $4, compared with $3 for helping.** The tournament's expected prize is (1/4)($32) = $8. Subtract the $4 entry cost to get $4.
2. **Sell the books. The hoodie's expected profit is $2, compared with $4 for the books.** The expected hoodie sale price is (1/2)($30) + (1/2)($10) = $20. Subtract the $18 purchase cost to get $2.

CHALLENGE

The expected profit is 40x + 8(1 - x) - 10 = 32x - 2. For the pack to earn at least $2 on average, 32x - 2 ≥ 2, so x ≥ 1/8. The smallest value is 1/8.

Accept equivalent fractions or decimals for expected values. In decisions, students should compare expected net gains, not just the largest possible prize.