LESSON MAKER PRO

Arc to Angle

Use arc length and radius to find radian measure.

HIGH SCHOOL · A2S.GM.A.34

A2S.GM.A.34 "Define the radian measure of an angle as the constant of proportionality of the length of an arc it intercepts to the radius of the circle; in particular, it is the length of the arc intercepted on the unit circle."

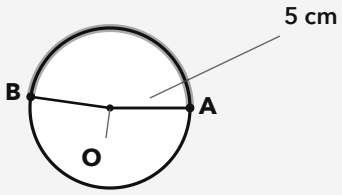
NAME _____ DATE _____

THE RADIAN RULE

An angle's radian measure is the length of its intercepted arc divided by the circle's radius: angle = arc length / radius. This ratio stays the same when the circle changes size. On a unit circle, the radius is 1, so the radian measure equals the intercepted arc length.

WORKED EXAMPLE

The highlighted arc is 15 cm long. Find the angle at O in radians.

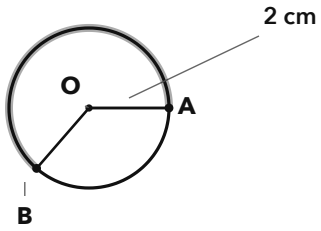


1. Use angle = arc length / radius.
2. Substitute: angle = 15 / 5.
3. Divide: angle = 3.

Answer: 3 radians

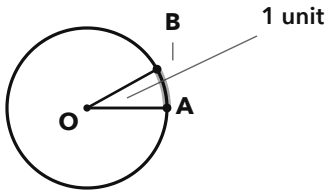
PRACTICE 6 problems

1 The highlighted arc is 8 cm long. Find the angle at O in radians.



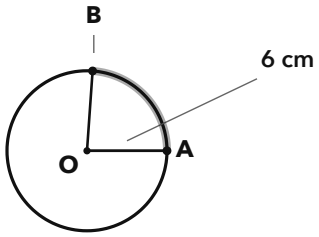
ANSWER _____

2 The highlighted arc is 1/2 unit long. Find the angle at O in radians.



ANSWER _____

3 The highlighted arc is 9 cm long. Find the angle at O in radians.



ANSWER _____

4 A circle has radius 7 cm. An angle measures 2 radians. Find its intercepted arc length.

ANSWER _____

5 An arc is 11 cm long and its central angle measures 1/2 radian. Find the circle's radius.

ANSWER _____

6 One circle has an arc of 10 cm and a radius of 8 cm. Another has an arc of 20 cm and a radius of 16 cm. Find both angles in radians. Do the angles have the same measure?

ANSWER _____

_____

APPLY IT Show your work

1 A runner travels 30 m along a circular track with a radius of 20 m. What angle, in radians, does the runner travel around the track's center?

WORK SPACE

ANSWER _____

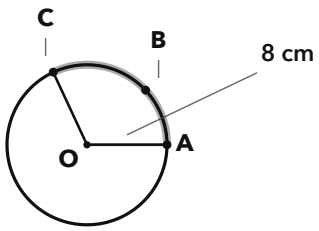
2 You turn a music app's circular dial. Your finger traces an arc of 8 mm at a distance of 12 mm from the dial's center. How far did you turn the dial, in radians?

WORK SPACE

ANSWER _____

CHALLENGE One step up

In the circle, arc AB is 6 cm long and arc BC is 10 cm long. Find the radian measure of angle AOC and the length of its intercepted arc on a unit circle. Explain why those two numbers agree.



WORK SPACE AND ANSWER

ANSWER KEY # Arc to Angle

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	The highlighted arc is 8 cm long. Find the angle at O in radians.	**4 radians**
2	The highlighted arc is 1/2 unit long. Find the angle at O in radians.	**1/2 radian**
3	The highlighted arc is 9 cm long. Find the angle at O in radians.	**3/2 radians**
4	A circle has radius 7 cm. An angle measures 2 radians. Find its intercepted arc length.	**14 cm**
5	An arc is 11 cm long and its central angle measures 1/2 radian. Find the circle's radius.	**22 cm**
6	One circle has an arc of 10 cm and a radius of 8 cm. Another has an arc of 20 cm and a radius of 16 cm. Find both angles in radians. Do the angles have the same measure?	**Both angles measure 5/4 radians. Yes, they have the same measure.**

PART 2 · APPLY IT

1. **3/2 radians** Divide the distance along the track by its radius: 30 / 20 = 3/2.
2. **2/3 radian** Divide the arc length by the radius: 8 / 12 = 2/3.

CHALLENGE

Angle AOC measures (6 + 10) / 8 = 2 radians. Its intercepted arc on a unit circle is 2 units long because arc length = angle × radius, and the unit circle's radius is 1.

Accept equivalent exact fractions and answers that omit the word “radians” when the prompt specifies the unit. In the challenge, accept any explanation that correctly uses the unit circle's radius of 1.