LESSON MAKER PRO

Figure Detective

Build figures and test their properties on coordinate grids.

HIGH SCHOOL · GDA.GM.G.29

GDA.GM.G.29 "Find patterns and relationships in figures including lines, triangles, quadrilaterals, and circles, using technology and other tools. a. Construct figures, using technology and other tools..."

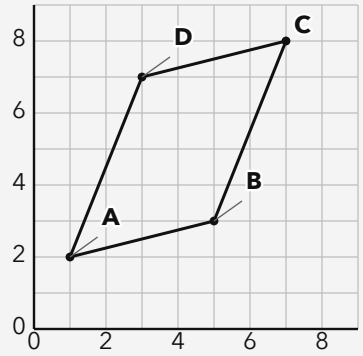
NAME _____ DATE _____

BUILD, THEN TEST

On a coordinate grid, subtract coordinates to find a side's horizontal and vertical changes. Matching changes can help you place a missing vertex, while squared side lengths can help you compare sides. To decide whether properties define a figure, ask whether a different figure could have those same properties.

WORKED EXAMPLE

Find the coordinates of D in the plotted figure. Use side vectors to test whether both pairs of opposite sides are parallel.

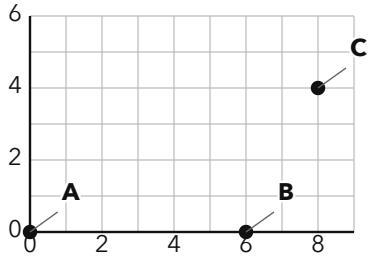


1. Read D from the grid: D = (3, 7).
2. From A to B and from D to C, the change is (4, 1).
3. From A to D and from B to C, the change is (2, 5).

Answer: D = (3, 7). Both pairs of opposite sides have matching vectors, so both pairs are parallel.

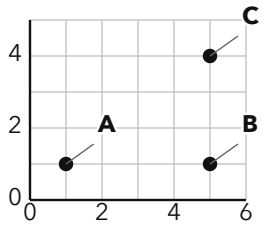
PRACTICE Begin here

1 Place D so ABCD has two pairs of parallel sides. Give D's coordinates and the change from A to D.



ANSWER _____

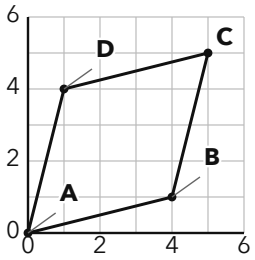
2 Place D so ABCD is a rectangle. Give D's coordinates. Is this rectangle also a square? Use side lengths to explain.



ANSWER _____

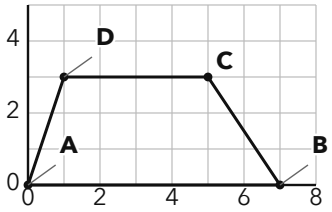
PRACTICE Continue

3 A student claims the plotted figure is a square because all four sides are equal. Test the claim using side lengths and the changes along AB and BC.



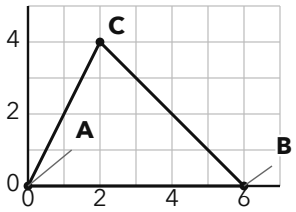
ANSWER _____

4 Does this figure disprove the claim that one pair of parallel sides is enough to define a parallelogram? Compare the changes along its opposite sides.



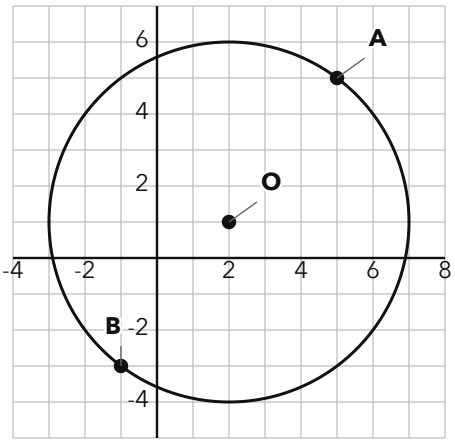
ANSWER _____

5 Use squared side lengths to test whether triangle ABC is isosceles.



ANSWER _____

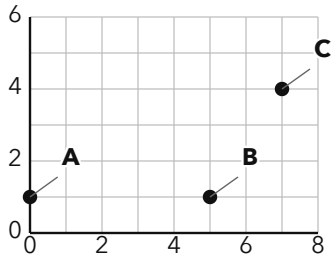
6 Is AB a diameter of the plotted circle? Find its length and use the midpoint of AB to justify your answer.



ANSWER _____

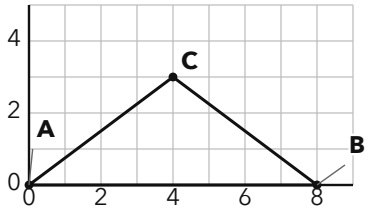
APPLY IT Show your work

1 A game designer plots three corners of a floating tile. Place D so ABCD is a parallelogram. The designer also wants a rhombus. Will this tile meet that goal? Compare adjacent side lengths.



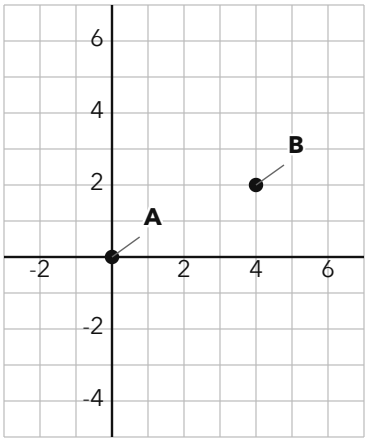
ANSWER _____

2 A skate park planner plots the three supports of a ramp frame. The frame must be an isosceles triangle. Do the plotted points meet that requirement? Find the frame's perimeter.



ANSWER _____

CHALLENGE One step up



A designer needs a square ABCD, with vertices named in order. The grid shows A and B. Give both possible pairs of coordinates for C and D, find the square's area, and explain why each construction makes a square.

WORK SPACE AND ANSWER

Figure Detective

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Place D so ABCD has two pairs of parallel sides. Give D's coordinates and the change from A to D.	**D = (2, 4); the change from A to D is (2, 4).**
2	Place D so ABCD is a rectangle. Give D's coordinates. Is this rectangle also a square? Use side lengths to explain.	**D = (1, 4). No; adjacent sides have lengths 4 and 3.**
3	A student claims the plotted figure is a square because all four sides are equal. Test the claim using side lengths and the changes along AB and BC.	**The claim is false. Each side has squared length 17, but AB = (4, 1) and BC = (1, 4) have dot product 8, not 0, so they do not form a right angle.**
4	Does this figure disprove the claim that one pair of parallel sides is enough to define a parallelogram? Compare the changes along its opposite sides.	**Yes. AB and DC are horizontal, but AD = (1, 3) and BC = (-2, 3) are not parallel.**
5	Use squared side lengths to test whether triangle ABC is isosceles.	**No. $AB^2 = 36$, $AC^2 = 20$, and $BC^2 = 32$, so no two sides are equal.**
6	Is AB a diameter of the plotted circle? Find its length and use the midpoint of AB to justify your answer.	**Yes. The midpoint of AB is (2, 1), the center O, and AB = 10 units.**

PART 2 · APPLY IT

1. **D = (2, 4). No; AB = 5 units and BC = √13 units, so the tile is not a rhombus.** Match the change from B to C, (2, 3), with the change from A to D. Then compare the lengths of adjacent sides AB and BC.
2. **Yes. AC = BC = 5 units, and the perimeter is 18 units.** Each sloping side has length 5 by the distance formula. Add those lengths to the base length of 8.

CHALLENGE

One construction is C = (2, 6), D = (-2, 4). The other is C = (6, -2), D = (2, -4). Both have area 20 square units: AB = (4, 2) has squared length 20, and each possible BC, (-2, 4) or (2, -4), has squared length 20 and a dot product of 0 with AB. In each construction, C = B + D - A makes opposite sides equal and parallel, so the equal perpendicular adjacent sides form a square.

Accept equivalent distance expressions, such as √20 instead of 2√5. For the construction items, students may justify parallel sides with matching coordinate changes or equal slopes.