LESSON MAKER PRO

Function Undo

Compose functions and test whether they are inverses.

HIGH SCHOOL · A2.F.2.2

A2.F.2.2 "Combine functions by composition and recognize that $g(x) = f^{-1}(x)$, the inverse function of f(x), if and only if f(g(x)) = g(f(x)) = x."

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THE BIG IDEA

To find f(g(x)), put the entire expression for g(x) in place of x in f. Two functions are inverses if both f(g(x)) and g(f(x)) equal x for every input where each composition is defined.

WORKED EXAMPLE

For f(x) = 13x + 17 and g(x) = (x - 17)/13, check whether g is the inverse of f.

1. f(g(x)) = 13((x - 17)/13) + 17
2. f(g(x)) = x - 17 + 17 = x
3. g(f(x)) = ((13x + 17) - 17)/13
4. g(f(x)) = 13x/13 = x

Answer: Yes. Both compositions equal x.

PRACTICE 6 problems

1 Let f(x) = 2x + 3 and g(x) = x - 4. Find f(g(x)).

ANSWER _____

2 Let $f(x) = 3x^2$ and g(x) = x + 2. Find g(f(x)) and f(g(x)).

ANSWER _____

3 Let f(x) = 4x - 6 and g(x) = (x + 6)/4. Find both compositions. Are f and g inverses?

ANSWER _____

4 Let f(x) = 2x + 6 and g(x) = x/2 + 3. Find both compositions. Are f and g inverses?

ANSWER _____

5 Let f(x) = (x - 4)/3. Find a function g that is the inverse of f, then state the result of each composition.

ANSWER _____

6 Let $f(x) = x^2 + 6$ for x >= 0 and g(x) = sqrt(x - 6) for x >= 6. Find both compositions and state the allowed inputs for each.

ANSWER _____

APPLY IT Show your work

1 A music app starts you with 24 credits. Each download costs 4 credits. Let f(d) = 24 - 4d give the credits left after d downloads, and let g(c) = (24 - c)/4 give the number of downloads made when c credits are left. Find f(g(8)) and g(f(2)). Explain what each result means.

WORK SPACE

ANSWER _____

2 A store takes $8 off a $32 hoodie, then adds 10% sales tax. Let f(x) = x - 8 apply the discount and g(x) = 11x/10 add the tax. Find g(f(32)). How much more would the hoodie cost if the store added tax before applying the discount?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Let f(x) = 2/(x - 3) + 4, where x is not 3. Find its inverse g, give the input excluded from g, and show that both compositions equal x. Explain why g excludes a different input than f.

WORK SPACE AND ANSWER

ANSWER KEY # Function Undo

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Let f(x) = 2x + 3 and g(x) = x - 4. Find f(g(x)).	**2x - 5**
2	Let f(x) = $3x^2$ and g(x) = x + 2. Find g(f(x)) and f(g(x)).	**g(f(x)) = $3x^2$ + 2; f(g(x)) = $3x^2$ + 12x + 12**
3	Let f(x) = 4x - 6 and g(x) = (x + 6)/4. Find both compositions. Are f and g inverses?	**f(g(x)) = x and g(f(x)) = x. Yes, they are inverses.**
4	Let f(x) = 2x + 6 and g(x) = x/2 + 3. Find both compositions. Are f and g inverses?	**f(g(x)) = x + 12 and g(f(x)) = x + 6. No, they are not inverses.**
5	Let f(x) = (x - 4)/3. Find a function g that is the inverse of f, then state the result of each composition.	**g(x) = 3x + 4; f(g(x)) = x and g(f(x)) = x**
6	Let f(x) = x^2 + 6 for x >= 0 and g(x) = sqrt(x - 6) for x >= 6. Find both compositions and state the allowed inputs for each.	**f(g(x)) = x for x >= 6; g(f(x)) = x for x >= 0**

PART 2 · APPLY IT

1. **f(g(8)) = 8 credits: when 8 credits remain, g(8) = 4 downloads and f(4) = 8 credits. g(f(2)) = 2 downloads: after 2 downloads, f(2) = 16 credits remain, and g(16) = 2 downloads.** g(8) = 4 downloads, so f(g(8)) = f(4) = 8 credits. Also, f(2) = 16 credits and g(16) = 2 downloads.
2. **The hoodie costs $26.40 with the discount first. Taxing first would cost $0.80 more.** g(f(32)) = g(24) = $26.40. Reversing the order gives f(g(32)) = f(35.20) = $27.20, a difference of $0.80.

CHALLENGE

g(x) = 3 + 2/(x - 4), where x is not 4. f(g(x)) = 2/(2/(x - 4)) + 4 = x for x not 4, and g(f(x)) = 3 + 2/(2/(x - 3)) = x for x not 3. The inverse excludes 4 because f never outputs 4.

Accept equivalent simplified forms in the composition problems. In the music-app problem, distinguish credits from downloads when interpreting the results.