LESSON MAKER PRO

Limits and Gaps

Find what a rational function approaches and where it is undefined.

HIGH SCHOOL · PC.F.1.4

PC.F.1.4 "Describe end behavior, asymptotic behavior, and points of discontinuity."

NAME _____ DATE _____

FACTOR FIRST

Factor the numerator and denominator and cancel all common factors, keeping the original denominator zeros excluded. At an excluded input, a nonzero reduced denominator gives a hole; a zero of the fully reduced denominator gives a vertical asymptote. When numerator and denominator have the same degree, their leading-coefficient ratio gives the horizontal asymptote and the value approached at both ends.

WORKED EXAMPLE

Use the function card to find the hole, asymptotes, and end behavior of f(x).

F(X) = NUMERATOR/DENOMINATOR

Numerator	Denominator
(x+9)(x-11)	(x-11)(x-6)

1. Cancel x-11, but keep x=11 excluded.
2. At x=11, (x+9)/(x-6)=20/5=4, so the hole is (11, 4).
3. The remaining denominator gives x=6. The original leading coefficients are both 1.

Answer: Hole: (11, 4). Vertical asymptote: x=6. Horizontal asymptote: y=1. As x approaches positive or negative infinity, f(x) approaches 1.

PRACTICE 4 problems

1 For f(x)=(x-1)(x+4)/((x-1)(x+2)), find the hole and vertical asymptote.

ANSWER _____

2 For f(x)=(3x+1)/(x-4), find the vertical and horizontal asymptotes.

ANSWER _____

3 As x approaches positive or negative infinity, what value does $f(x)=(2x^2+1)/(x^2+9)$ approach?

ANSWER _____

4 For $f(x)=(x+3)/(x^2-9)$, find each discontinuity. Is it a hole or a vertical asymptote?

ANSWER _____

APPLY IT Show your work

1 A music studio charges $24 plus $5 per hour. For $x>0$ hours, the average cost per hour is $C(x)=(5x+24)/x$. Describe $C(x)$ as x approaches 0 from the right and as x grows without bound.

WORK SPACE

ANSWER _____

2 An esports site's weekly viewers, in thousands, are modeled by $V(x)=(x^2-16)/(x-4)$, where x is weeks since launch and $x\geq 0$. What happens at week 4, and what value does $V(x)$ approach near week 4?

WORK SPACE

ANSWER _____

CHALLENGE One step up

For $f(x)=(x^2-25)/(x^2-3x-10)$, find the hole, vertical asymptote, and end behavior. Explain what happens as x approaches the vertical asymptote from each side.

WORK SPACE AND ANSWER

ANSWER KEY

Limits and Gaps

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For f(x)=(x-1)(x+4)/((x-1)(x+2)), find the hole and vertical asymptote.	**Hole: (1, 5/3). Vertical asymptote: x=-2.**
2	For f(x)=(3x+1)/(x-4), find the vertical and horizontal asymptotes.	**Vertical asymptote: x=4. Horizontal asymptote: y=3.**
3	As x approaches positive or negative infinity, what value does $f(x)=(2x^2+1)/(x^2+9)$ approach?	**2**
4	For $f(x)=(x+3)/(x^2-9)$, find each discontinuity. Is it a hole or a vertical asymptote?	**Hole at x=-3, at (-3, -1/6). Vertical asymptote at x=3.**

PART 2 · APPLY IT

1. **As x approaches 0 from the right, C(x) increases without bound. As x grows without bound, C(x) approaches $5 per hour.** Rewrite C(x) as 5+24/x. The term 24/x grows without bound near 0 from the right and approaches 0 as x grows.
2. **There is a hole at (4, 8). Near week 4, the model approaches 8 thousand viewers.** Factor the numerator as (x-4)(x+4). The expression is undefined at x=4, but x+4 approaches 8.

CHALLENGE

Factor to get (x-5)(x+5)/((x-5)(x+2)). The hole is (5, 10/7), and the vertical asymptote is x=-2. As x approaches negative or positive infinity, f(x) approaches 1. As x approaches -2 from the left, f(x) approaches negative infinity; from the right, it approaches positive infinity.

Accept equivalent limit notation. A hole's x-coordinate alone is sufficient for practice problem 4, which asks for the discontinuity and its type.