

Triangle Circle Builders

Locate the two centers that create a triangle's special circles.

HIGH SCHOOL • MA.912.GR.5.3

MA.912.GR.5.3 "Construct the inscribed and circumscribed circles of a triangle."

NAME _____

DATE _____

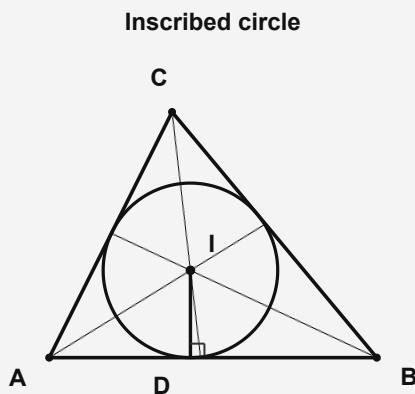
You will need: Pencil; Compass; Straightedge

TWO IMPORTANT CENTERS

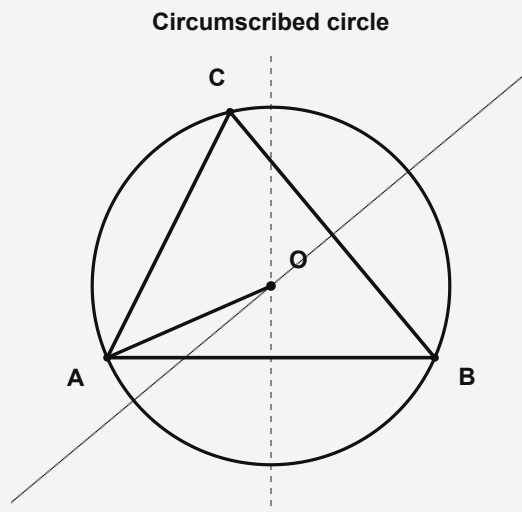
The angle bisectors of a triangle meet at the incenter. Use the incenter's perpendicular distance to a side as the radius of the inscribed circle. The perpendicular bisectors of the sides meet at the circumcenter, which is the center of the circumscribed circle.

WORKED EXAMPLE

Follow the two constructions for triangle ABC. Dotted lines show bisectors; the solid radius sets the compass width.



I: incenter; ID: perpendicular radius



O: circumcenter; radius to a vertex

1. Bisect two angles with compass arcs; their intersection is I. Drop perpendicular ID to AB, then draw the circle centered at I with radius ID.
2. Construct perpendicular bisectors of two sides using equal-radius arcs from their endpoints. Their intersection is O. Draw the circle centered at O through A.

Answer: The inscribed circle has center I and radius ID. The circumscribed circle has center O and radius OA.

PRACTICE 4 problems

- 1** An inscribed circle touches all three sides of a triangle. Its center is the intersection of which lines?

ANSWER _____

- 2** A circle passes through all three vertices of a triangle. Its center is the intersection of which lines?

ANSWER _____

- 3** In right triangle XYZ, $\angle Y$ is 90° . The circumcenter is the midpoint of which side?

ANSWER _____

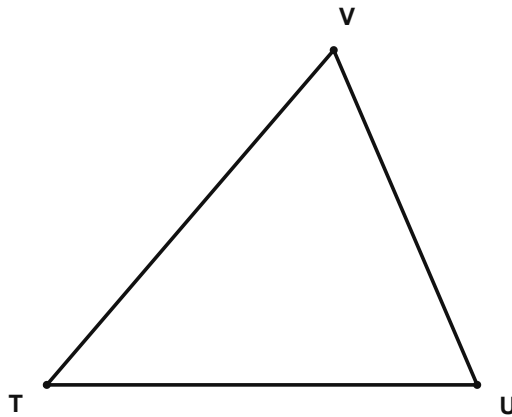
- 4** An incenter is 4 cm from each side of its triangle. What is the radius of the inscribed circle?

ANSWER _____

APPLY IT Show your work

- 1** Use a compass and straightedge to construct an inscribed circle in the esports logo TUV. Leave the angle-bisector arcs and perpendicular radius visible. Name the center and explain the compass width.

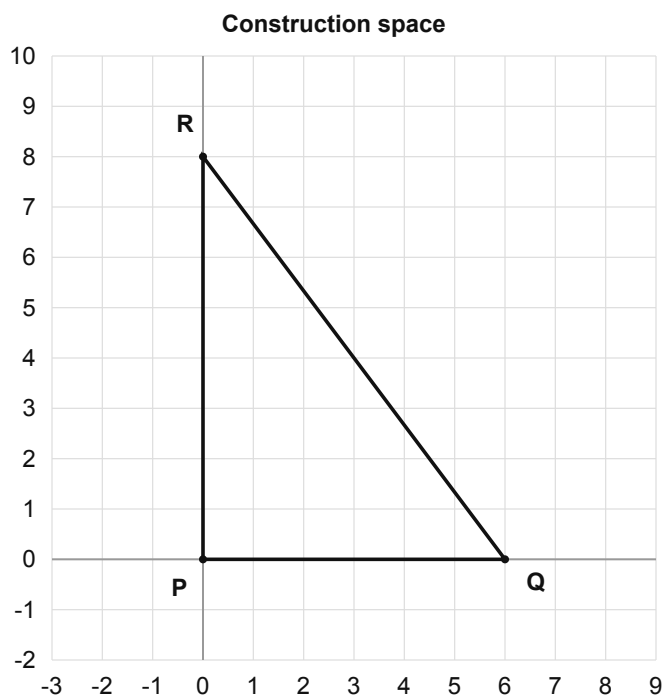
Construction space



Leave your construction arcs visible.

ANSWER _____

- 2 Sensors are $P(0,0)$, $Q(6,0)$ and $R(0,8)$. Use a compass and straightedge to construct their circumcircle on the grid. Leave the perpendicular bisectors visible. State the center and radius.



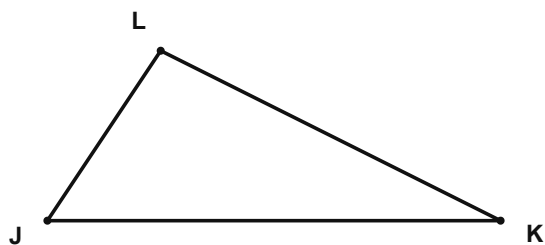
Leave your construction arcs visible.

ANSWER _____

CHALLENGE One step up

Use a compass and straightedge to construct both circles for obtuse triangle JKL. Keep the bisectors and radius segments visible. State whether each center is inside or outside the triangle and explain.

Construction space



Leave your construction arcs visible.

PRACTICE 1

An inscribed circle touches all three sides of a triangle. Its center is the intersection of which lines?

the three internal angle bisectors

PRACTICE 2

A circle passes through all three vertices of a triangle. Its center is the intersection of which lines?

the three perpendicular bisectors of the sides

PRACTICE 3

In right triangle XYZ, $\angle Y$ is 90° . The circumcenter is the midpoint of which side?

XZ

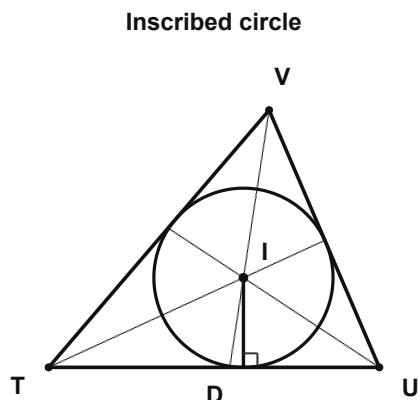
PRACTICE 4

An incenter is 4 cm from each side of its triangle. What is the radius of the inscribed circle?

4 cm

APPLY IT 1

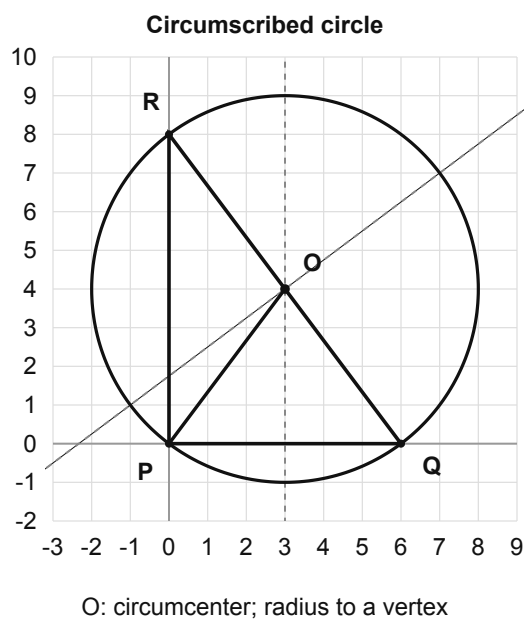
Construct the incenter. Set the compass width to the perpendicular distance from the incenter to any side. Angle bisectors locate the incenter. Its equal perpendicular distances to the three sides are the inscribed circle's radius.



I: incenter; ID: perpendicular radius

APPLY IT 2

Center: (3, 4); radius: 5 units Triangle PQR is right, so the circumcenter is the midpoint of hypotenuse QR. The midpoint of Q(6, 0) and R(0, 8) is (3, 4), and its distance to P is 5 units.

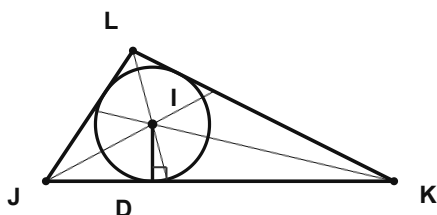


CHALLENGE

The incenter is inside the triangle because angle bisectors meet inside every triangle. The circumcenter is outside the triangle because an obtuse triangle has its perpendicular bisectors intersect outside.

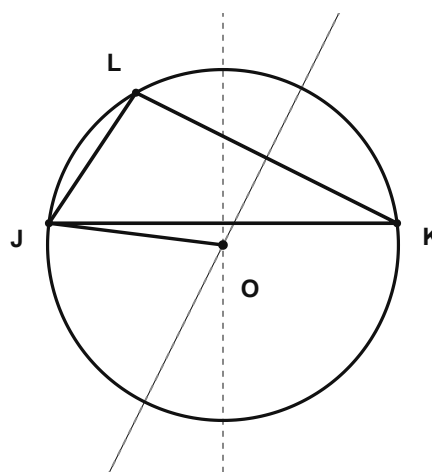
Accept "angle bisectors" and "perpendicular bisectors of the sides." For the sensor problem, accept (3, 4) and 5 units in either order if clearly labeled. Require actual circles and construction lines, not just center names. For angle bisectors, first swing an arc across both sides, then intersect equal-radius arcs from those two crossings. For a side's perpendicular bisector, use equal-radius arcs from the endpoints above and below the side. The incenter radius must be perpendicular to a side; a circumcircle radius reaches a vertex. All circle circumferences must fit on the paper. The teacher panels separate the two constructions for clarity; students may draw both on one triangle.

Inscribed circle



I: incenter; ID: perpendicular radius

Circumscribed circle



O: circumcenter; radius to a vertex