

Unit Circle Beyond

Use coterminal angles and unit-circle coordinates.

HIGH SCHOOL • HSF-TF.A

CCSS.Math.Content.HSF-TF.A "Extend the domain of trigonometric functions using the unit circle"

NAME _____

DATE _____

EXTEND THE CIRCLE

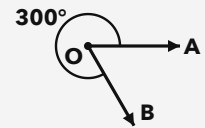
On the unit circle, the point at angle θ has coordinates $(\cos(\theta), \sin(\theta))$. Add or subtract 2π to replace a negative or large angle with a coterminal angle. Then use the coordinates to find trig values, including reciprocal functions when they are defined.

WORKED EXAMPLE

The terminal side in the figure is coterminal with $-\pi/3$. Find $\sec(-\pi/3)$.

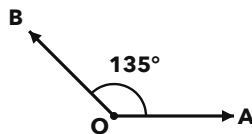
1. Add 2π : $-\pi/3 + 2\pi = 5\pi/3$.
2. The figure shows the same terminal side as $5\pi/3$.
3. At $5\pi/3$, $\cos(5\pi/3) = 1/2$.
4. $\sec(-\pi/3) = 1/(1/2) = 2$.

Answer: 2



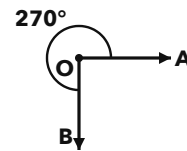
PRACTICE 6 problems

- 1** The angle in the figure is $3\pi/4$. Find $\sin(3\pi/4)$.



ANSWER _____

- 2** The angle in the figure is coterminal with $-\pi/2$. Find $\cos(-\pi/2)$.



ANSWER _____

- 3** Find $\tan(7\pi/6)$.

ANSWER _____

- 4** Find $\csc(-5\pi/4)$.

ANSWER _____

5 Find $\sin(11\pi/6)$.

ANSWER _____

6 Find $\cot(3\pi)$. State whether it is undefined.

ANSWER _____

APPLY IT Show your work

1 A student programs a small animation of a point moving on a unit circle. The point starts at $(1, 0)$ and moves counterclockwise $7\pi/6$ radians. What is the point's vertical coordinate relative to the center?

WORK SPACE

ANSWER _____

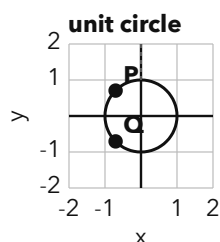
2 In a music app, a circular dial begins at $(1, 0)$ on a unit circle. The dial turns clockwise $5\pi/4$ radians. What are the coordinates of its final position?

WORK SPACE

ANSWER _____

CHALLENGE One step up

The grid shows points P and Q on the unit circle. Each has x-coordinate $-\sqrt{2}/2$. For $-2\pi \leq \theta \leq 2\pi$, list every value of θ for which $\cos(\theta) = -\sqrt{2}/2$. Explain how P and Q help you find the values.



WORK SPACE AND ANSWER

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	The angle in the figure is $3\pi/4$. Find $\sin(3\pi/4)$.	$\sqrt{2}/2$
2	The angle in the figure is coterminal with $-\pi/2$. Find $\cos(-\pi/2)$.	0
3	Find $\tan(7\pi/6)$.	$\sqrt{3}/3$
4	Find $\csc(-5\pi/4)$.	$\sqrt{2}$
5	Find $\sin(11\pi/6)$.	$-1/2$
6	Find $\cot(3\pi)$. State whether it is undefined.	undefined

PART 2 • APPLY IT

1. **$-1/2$** The vertical coordinate is $\sin(7\pi/6)$. The unit-circle y-coordinate at $7\pi/6$ is $-1/2$.
2. **$(-\sqrt{2}/2, \sqrt{2}/2)$** A clockwise turn of $5\pi/4$ is an angle of $-5\pi/4$, which is coterminal with $3\pi/4$. The unit-circle coordinates at $3\pi/4$ are $(-\sqrt{2}/2, \sqrt{2}/2)$.

CHALLENGE

$\theta = -5\pi/4, -3\pi/4, 3\pi/4, 5\pi/4$. Points P and Q give the two angles in one rotation, $3\pi/4$ and $5\pi/4$. Subtracting 2π gives the two negative coterminal angles.

Accept equivalent exact forms, including $1/\sqrt{3}$ for $\sqrt{3}/3$. For the challenge, accept the four values in any order if the explanation connects the shared x-coordinate to cosine and coterminal angles.