

Area Under Curves

Use definite integrals to measure regions on a graph.

HIGH SCHOOL · MA.912.C.5.5

MA.912.C.5.5 "Find the area between a curve and the x-axis or between two curves by using definite integrals."

NAME _____

DATE _____

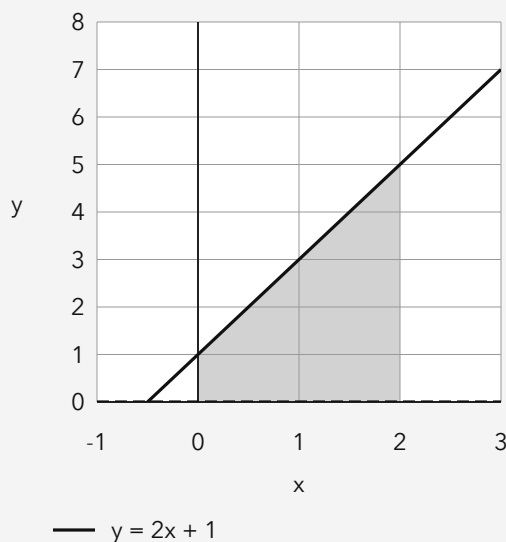
You will need: Pencil; Ruler

FIND THE REGION'S AREA

For a region above the x-axis, area = $\int$ (top function) dx over the stated interval. For a region between two curves, subtract bottom from top. If a curve crosses the x-axis, split the integral where it crosses so every piece of area is positive.

WORKED EXAMPLE

Find the area of the shaded region between $y = 2x + 1$ and the x-axis from $x = 0$ to $x = 2$.

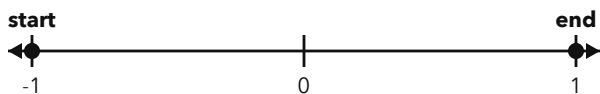


1. $A = \int_0^2 (2x + 1) dx$.
2. An antiderivative is $x^2 + x$.
3. $A = (2^2 + 2) - (0^2 + 0) = 6$.

Answer: 6 square units

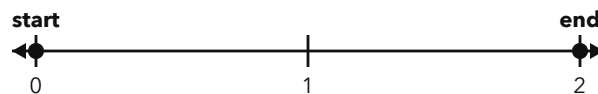
PRACTICE 6 problems

- 1** The number line shows the interval. Find the area between $y = x + 3$ and the x-axis.



ANSWER _____

- 2** The number line shows the interval. Find the area between $y = 2 - x$ and the x-axis.



ANSWER _____

- 3** Find the area between $y = x^2 - 1$ and the x-axis from $x = -1$ to $x = 1$.

ANSWER _____

- 4** Find the area between $y = 3x$ and $y = x^2$ from $x = 0$ to $x = 3$.

ANSWER _____

- 5** Find the total area between $y = x^3 - x$ and the x-axis from $x = -1$ to $x = 1$.

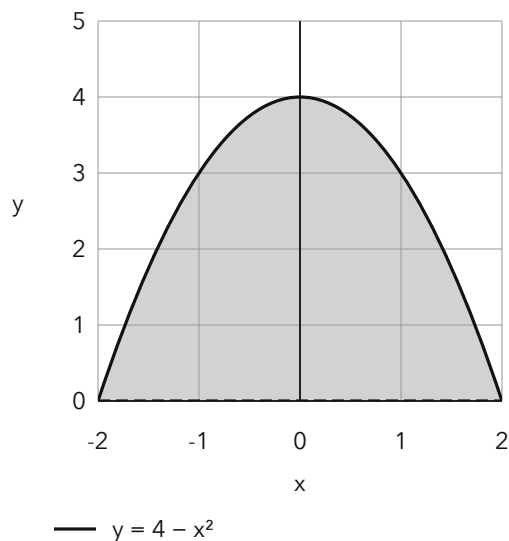
ANSWER _____

- 6** Find the area between $y = 9 - x^2$ and the x-axis from $x = -3$ to $x = 3$.

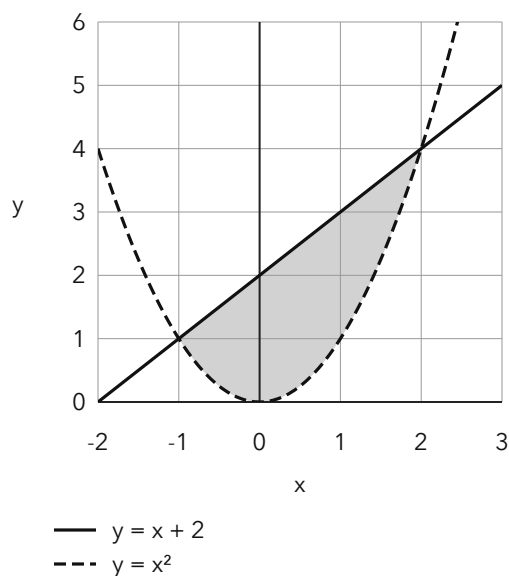
ANSWER _____

APPLY IT Show your work

- 1** A skatepark designer models the curved top edge of a ramp by the parabola in the graph. The ground is the x -axis. Find the area under the curved edge and above the ground, in square coordinate units.

**REASONING AND ANSWER**

- 2** An animation club uses the two curves in the graph as the boundaries of a logo region. Find the area of the region enclosed between the curves.

**REASONING AND ANSWER**

CHALLENGE One step up

Find the total area between $y = x^2 - 2x$ and the x-axis from $x = 0$ to $x = 3$. Show where you split the integral and explain why.

WORK SPACE AND ANSWER

PRACTICE 1**6 square units****PRACTICE 2****2 square units****PRACTICE 3** **$4/3$ square units****PRACTICE 4** **$9/2$ square units****PRACTICE 6****36 square units****APPLY IT 1**

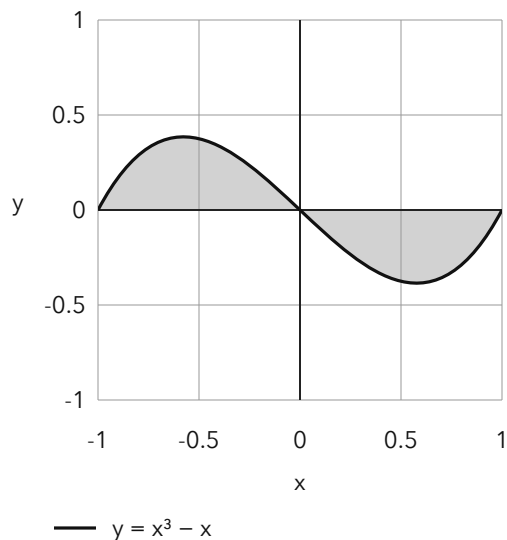
$32/3$ square coordinate units Use the x-intercepts shown, -2 and 2. Compute $\int_{-2}^2 (4 - x^2) dx = 32/3$.

APPLY IT 2

$9/2$ square coordinate units The curves meet at $x = -1$ and $x = 2$, and $y = x + 2$ is on top between them. Compute $\int_{-1}^2 (x + 2 - x^2) dx = 9/2$.

PRACTICE 5

1/2 square unit. Split at $x = 0$: $\int_{-1}^0 (x^3 - x) dx - \int_0^1 (x^3 - x) dx = 1/4 + 1/4$.



CHALLENGE

8/3 square units. Split at $x = 2$ because the curve crosses the x-axis there. The area is $-\int_0^2 (x^2 - 2x) dx + \int_2^3 (x^2 - 2x) dx = 8/3$.

Accept equivalent definite-integral setups and equivalent exact forms. For geometric area, accept only nonnegative results, with intervals split at x-intercepts when needed. Shading denotes geometric area, not signed integral. Dashed curve styles in the two-function illustration distinguish the functions; they do not mean excluded area boundaries.

