

Around Again

Use unit-circle turns to find sine and cosine.

HIGH SCHOOL • HSF-TF.A.2

CCSS.Math.Content.HSF-TF.A.2 "Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle."

NAME _____

DATE _____

ONE CIRCLE, ANY ANGLE

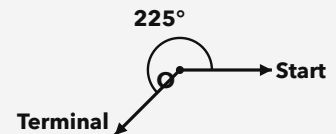
On the unit circle, the terminal point for an angle θ is $(\cos \theta, \sin \theta)$. Radians measure rotation around the circle, and one full counterclockwise turn is 2π radians. Adding or subtracting 2π keeps the same terminal point.

WORKED EXAMPLE

On the unit circle, use the angle shown to find its radian measure, terminal point, cosine, and sine.

- $225^\circ \times \pi/180^\circ = 5\pi/4$.
- The angle is 45° past π in Quadrant III.
- The terminal point is $(-\sqrt{2}/2, -\sqrt{2}/2)$.

Answer: $5\pi/4$; terminal point $(-\sqrt{2}/2, -\sqrt{2}/2)$; $\cos \theta = -\sqrt{2}/2$; $\sin \theta = -\sqrt{2}/2$



PRACTICE 4 problems

- 1** Find the terminal point for $\theta = -\pi/2$ on the unit circle.

ANSWER _____

- 2** For $\theta = 13\pi/6$, find the coterminal angle in $[0, 2\pi)$ and the terminal point.

ANSWER _____

- 3** A terminal point is $(0, -1)$. Give its angle in the interval $[-2\pi, 0]$.

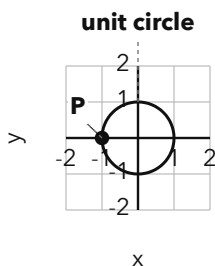
ANSWER _____

- 4** Find $\cos \theta$ and $\sin \theta$ for $\theta = 7\pi/3$.

ANSWER _____

APPLY IT Show your work

- 1** A game designer places a sprite at point P on the unit circle. The sprite rotates counterclockwise from $(1, 0)$ and stops when it first reaches P. What is the rotation in radians, and what are $\cos \theta$ and $\sin \theta$?



ANSWER _____

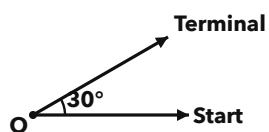
- 2** A music app's turntable marker starts at $(1, 0)$ and rotates $17\pi/6$ radians counterclockwise. How many complete rotations occur, and where does the marker end?

WORK SPACE

ANSWER _____

CHALLENGE One step up

The figure shows a terminal side on the unit circle. Explain why $\sin(\theta + 2\pi k)$ and $\cos(\theta + 2\pi k)$ have the same values as $\sin \theta$ and $\cos \theta$ for every integer k . Include the sine and cosine values for the shown angle.



WORK SPACE AND ANSWER

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the terminal point for $\theta = -\pi/2$ on the unit circle.	(0, -1)
2	For $\theta = 13\pi/6$, find the coterminal angle in $[0, 2\pi)$ and the terminal point.	$\pi/6$; $(\sqrt{3}/2, 1/2)$
3	A terminal point is (0, -1). Give its angle in the interval $[-2\pi, 0]$.	$-\pi/2$
4	Find $\cos \theta$ and $\sin \theta$ for $\theta = 7\pi/3$.	$\cos \theta = 1/2$; $\sin \theta = \sqrt{3}/2$

PART 2 · APPLY IT

1. **π ; $\cos \theta = -1$; $\sin \theta = 0$** Point P is at (-1, 0), halfway around the unit circle from (1, 0). Half of 2π is π .
2. **1 complete rotation; terminal point $(-\sqrt{3}/2, 1/2)$** $17\pi/6 = 2\pi + 5\pi/6$, so one full turn is removed. The terminal point for $5\pi/6$ is $(-\sqrt{3}/2, 1/2)$.

CHALLENGE

The shown 30° angle is $\pi/6$, so $\cos \theta = \sqrt{3}/2$ and $\sin \theta = 1/2$. Adding $2\pi k$ adds whole turns, which return the terminal side to the same point, so the cosine and sine values stay $\sqrt{3}/2$ and $1/2$.

Accept equivalent radical forms and ordered-pair formatting. For the challenge, accept any explanation that states that whole 2π turns return to the same terminal point.