

When Growth Takes Over

Compare linear, polynomial, and exponential growth.

HIGH SCHOOL • HSF-LE.A.3

CCSS.Math.Content.HSF-LE.A.3 "Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function."

NAME _____

DATE _____

THE BIG IDEA

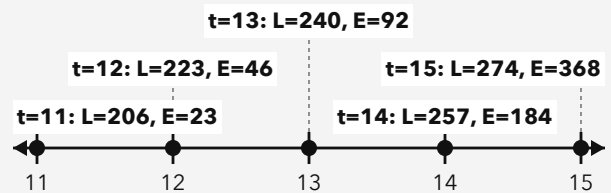
As inputs become large, an exponential quantity eventually exceeds a linear quantity. Compare matched inputs in a table or graph to locate when the exponential value becomes larger.

WORKED EXAMPLE

Use the timeline. At what first marked value of t does E exceed L ?

1. At $t=14$, $L=257$ and $E=184$, so E has not exceeded L .
2. At $t=15$, $L=274$ and $E=368$, so E exceeds L .
3. The first marked value with $E > L$ is 15.

Answer: $t = 15$



PRACTICE 6 problems

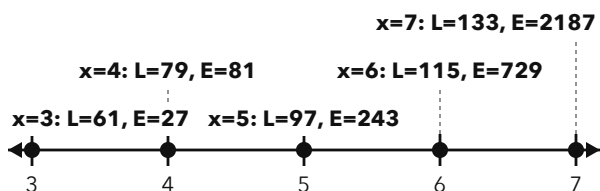
- 1** For $f(x) = 7x + 1$ and $g(x) = 3^x$, find the first positive whole-number value of x for which $g(x) > f(x)$.

ANSWER _____

- 2** For $p(x) = 5x^3$ and $q(x) = 4^x$, find the first positive whole-number value of x for which $q(x) > p(x)$.

ANSWER _____

- 3** Use the timeline. At what first marked input does E exceed L ?



ANSWER _____

- 4** For $a(x) = 20x + 30$ and $b(x) = 7^x$, find the first positive whole-number value of x for which $b(x) > a(x)$.

ANSWER _____

- 5** For $r(x) = 9x^3$ and $s(x) = 4^x$, which function is larger at $x = 6$? Is $x = 6$ the first positive whole-number input where $s(x) > r(x)$?

ANSWER _____

- 6** For $m(x) = 10x + 8$ and $n(x) = 3^x$, give two whole-number inputs after $n(x)$ first exceeds $m(x)$.

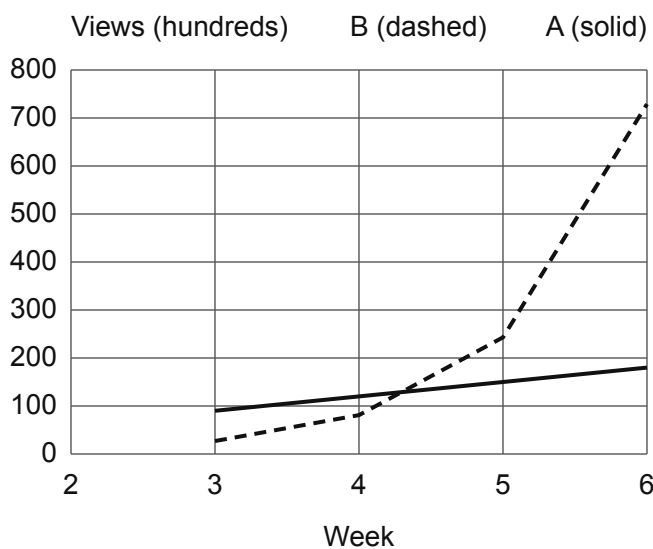
ANSWER _____

APPLY IT Show your work

- 1** Two video channels track views, in hundreds, at the beginning of each week. Channel A grows linearly, and Channel B grows exponentially. Use the table and the accompanying graph; the graph connects the shown weekly observations. During which first shown week does Channel B have more views than Channel A?

VIDEO VIEWS, IN HUNDREDS

Week	Channel A	Channel B
3	90	27
4	120	81
5	150	243
6	180	729



ANSWER _____

2

A gaming clip earns 40 new views each day and begins with 10 views, so its views after d days are $40d + 10$. A shared clip has 3^d views after d days. On what first positive whole-number day does the shared clip have more views, and how many views does each clip have that day?

WORK SPACE

ANSWER _____**CHALLENGE** One step up

For $h(x) = 6x^3$ and $j(x) = 4^x$, find the first positive whole-number value of x for which $j(x) > h(x)$. Then compare the two functions at $x = 6$ and $x = 7$. Explain how those comparisons support the claim that exponential growth eventually takes over.

PRACTICE 1

For $f(x) = 7x + 1$ and $g(x) = 3^x$, find the first positive whole-number value of x for which $g(x) > f(x)$.

$x = 3$

PRACTICE 2

For $p(x) = 5x^3$ and $q(x) = 4^x$, find the first positive whole-number value of x for which $q(x) > p(x)$.

$x = 5$

PRACTICE 3

Use the timeline. At what first marked input does E exceed L?

$x = 4$

PRACTICE 4

For $a(x) = 20x + 30$ and $b(x) = 7^x$, find the first positive whole-number value of x for which $b(x) > a(x)$.

$x = 3$

PRACTICE 5

For $r(x) = 9x^3$ and $s(x) = 4^x$, which function is larger at $x = 6$? Is $x = 6$ the first positive whole-number input where $s(x) > r(x)$?

$s(x)$ is larger at $x = 6$. Yes, $x = 6$ is the first positive whole-number input where $s(x) > r(x)$.

PRACTICE 6

For $m(x) = 10x + 8$ and $n(x) = 3^x$, give two whole-number inputs after $n(x)$ first exceeds $m(x)$.

For example, $x = 5$ and $x = 6$. The first crossing is $x = 4$, so the requested inputs must be greater than 4.

APPLY IT 1

Week 5. Channel B has 243 hundred views, while Channel A has 150 hundred views.

Compare the two channel values in each row. Channel B is lower in weeks 3 and 4, then becomes higher in week 5.

APPLY IT 2

Day 5. The gaming clip has 210 views, and the shared clip has 243 views. Check positive

whole-number days until 3^d exceeds $40d + 10$. At day 5, 243 is greater than 210.

CHALLENGE

The first value is $x = 5$, because $j(5) = 1024$ and $h(5) = 750$. At $x = 6$, $j(6) = 4096$ and $h(6) = 1296$. At $x = 7$, $j(7) = 16384$ and $h(7) = 2058$. The exponential value is larger after $x = 5$, and the gap grows at $x = 6$ and $x = 7$.

Accept equivalent comparison statements, such as “the exponential function is larger.” For first-input questions, accept only the earliest positive whole-number input that makes the exponential value greater. Practice 6 asks for any two whole-number inputs strictly after the first crossing; accept any two distinct inputs greater than 4. The connected graph shows the listed weekly values; use the table for exact comparisons.