

Triangle Distance Detectives

Use sine and cosine relationships to solve any triangle.

HIGH SCHOOL • HSG-SRT.D.10

CCSS.Math.Content.HSG-SRT.D.10 "(+) Prove the Laws of Sines and Cosines and use them to solve problems."

NAME _____

DATE _____

You will need: Pencil; Ruler

TWO TRIANGLE LAWS

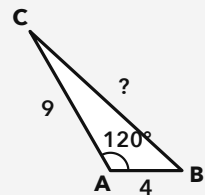
For a triangle with sides a , b , and c opposite angles A , B , and C , use $a/\sin A = b/\sin B = c/\sin C$ when you know an angle-side pair. Use $c^2 = a^2 + b^2 - 2ab \cos C$ when you know two sides and their included angle. Both laws can be developed by dropping an altitude to split an oblique triangle into right triangles.

WORKED EXAMPLE

Find side BC in the triangle.

1. Let $x = BC$.
2. $x^2 = 4^2 + 9^2 - 2(4)(9)\cos 120^\circ$.
3. $x^2 = 16 + 81 + 36 = 133$.
4. $x = \sqrt{133}$.

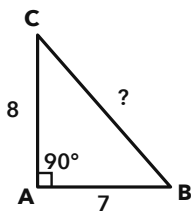
Answer: $\sqrt{133}$



PRACTICE 6 problems

Show your calculation or reasoning. Include units when needed.

- 1 Find the length of BC in the triangle.



ANSWER _____

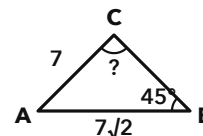
- 2 In triangle ABC , $a = 8$, $A = 30^\circ$, and $B = 45^\circ$. Find b .

ANSWER _____

- 3** In triangle ABC, $a = 10$, $b = 10$, and $C = 90^\circ$. Find c .

ANSWER _____

- 4** Find angle C in the triangle.



ANSWER _____

- 5** In triangle ABC, $a = 7$, $b = 11$, and $c = \sqrt{93}$. Find angle C.

ANSWER _____

- 6** For triangle ABC, write the Law of Cosines equation that relates c to a , b , and angle C. Prove it: place $C = (0, 0)$, $A = (b, 0)$, and $B = (a \cos C, a \sin C)$. Apply the distance formula to AB and simplify using $\sin^2 C + \cos^2 C = 1$. Then prove the Law of Sines: express the same triangle area K using each side as a base and its corresponding altitude. Obtain $2K = bc \sin A = ac \sin B = ab \sin C$; divide by abc and take reciprocals. Explain your steps.

ANSWER _____

APPLY IT Show your work

- 1** At a campus festival, two friends leave the student center on paths that form a 60° angle. One friend walks 12 m to a photo booth, and the other walks 15 m to a snack stand. How far apart are the photo booth and snack stand? Give an exact answer.

- 2** A triangular robotics course has a 30° angle at the start and a 45° angle at the first checkpoint. The side opposite the start angle is 14 m. Find the length of the side opposite the checkpoint angle.

CHALLENGE One step up

In triangle ABC, $A = 30^\circ$, $a = 6$, and $b = 6\sqrt{3}$. This is an SSA situation. How many different triangles are possible, and what are the possible values of c ?

PRACTICE 2

$8\sqrt{2}$

PRACTICE 3

$10\sqrt{2}$

PRACTICE 5

60°

PRACTICE 6

$c^2 = a^2 + b^2 - 2ab \cos C$. With the stated coordinates, $c^2 = (b - a \cos C)^2 + (a \sin C)^2 = b^2 - 2ab \cos C + a^2(\cos^2 C + \sin^2 C) = a^2 + b^2 - 2ab \cos C$. For the area proof, the altitude to side c is $b \sin A$; to side a it is $c \sin B$; to side b it is $a \sin C$. Thus $2K = bc \sin A = ac \sin B = ab \sin C$. Dividing by abc gives $\sin A/a = \sin B/b = \sin C/c$; reciprocals give $a/\sin A = b/\sin B = c/\sin C$. All three angles are between 0° and 180° , so their sines are positive.

APPLY IT 1

$3\sqrt{21}$ m Use the Law of Cosines: $d^2 = 12^2 + 15^2 - 2(12)(15)\cos 60^\circ$. Since $\cos 60^\circ = 1/2$, $d^2 = 144 + 225 - 180 = 369 - 180 = 189$. Therefore $d = \sqrt{189} = 3\sqrt{21}$ m.

APPLY IT 2

$14\sqrt{2}$ m Use the Law of Sines: $b/\sin 45^\circ = 14/\sin 30^\circ$. Thus $b = 14\sqrt{2}$ m.

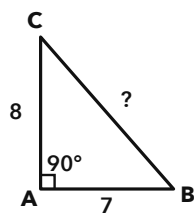
CHALLENGE

Two triangles are possible. The possible values of c are 6 and 12. By the Law of Sines, $\sin B = (6\sqrt{3})(1/2)/6 = \sqrt{3}/2$, giving $B = 60^\circ$ or 120° . Thus $C = 90^\circ$ or 30° , and $c = 12 \sin C$ gives 12 or 6.

Accept equivalent exact radical forms, such as $\sqrt{189}$ before simplification for the first word problem. For the challenge, accept both possible lengths only if the student identifies that two triangles are possible.

PRACTICE 1

$\sqrt{113}$



PRACTICE 4

90° . $\sin C / (7\sqrt{2}) = \sin 45^\circ / 7$, so $\sin C = 1$ and $C = 90^\circ$.

