

Move It Precisely

Develop geometric definitions of reflection, rotation, and translation.

HIGH SCHOOL • HSG-CO.A.4

CCSS.Math.Content.HSG-CO.A.4 "Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments."

NAME _____

DATE _____

A RULE MUST LOCATE EVERY IMAGE POINT

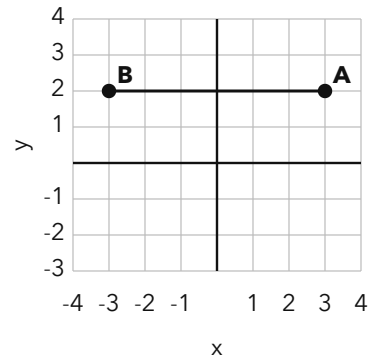
A precise transformation rule specifies how EVERY point moves. Use distances, angles, circles, and perpendicular or parallel segments to define its image. Include direction and fixed points when needed. Coordinates can test a geometric rule, but coordinate formulas alone do not explain the defining geometry.

WORKED EXAMPLE

$A(3,2)$ and $B(-3,2)$ lie on opposite sides of the y -axis. Explain why B is the reflection of A in that axis.

1. AB is horizontal, so it meets the vertical y -axis at a right angle.
2. The midpoint $(0,2)$ is on the y -axis. Thus the axis is the perpendicular bisector of AB .

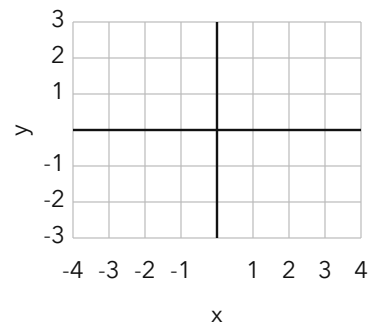
Answer: B is across the axis at the same perpendicular distance, 3 units.



PRACTICE 4 problems

- 1 Reflect $X(2,-1)$ across the y -axis. Plot X and X' , draw XX' , and mark its midpoint M . Use perpendicularity and equal lengths XM , MX' to justify the image.

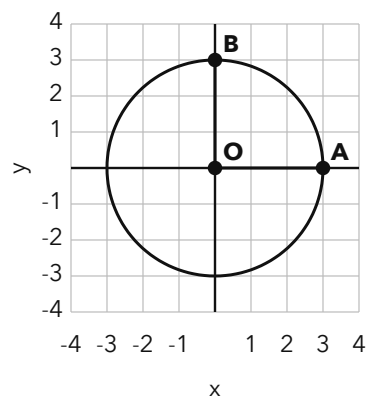
ANSWER _____



- 2** Develop a definition of reflection in any line ℓ . State the relationship between ℓ and the segment joining a point X off the line to its image X' . Also say what happens when X lies ON ℓ .

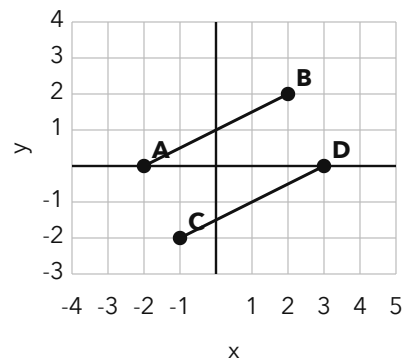
ANSWER _____

- 3** In the diagram, $OA = OB = 3$ and the counterclockwise angle AOB is 90° . Explain why B is a 90° counterclockwise rotation of A about O . Then define a rotation about O through ANY fixed directed angle θ , including the image of O .



ANSWER _____

- 4** $A(-2,0)$ maps to $B(2,2)$; $C(-1,-2)$ maps to $D(3,0)$. Compare the directed segments AB and CD . Develop a definition of translation by this displacement that applies to EVERY point X .



ANSWER _____

APPLY IT Show your work

- 1** A designer claims that any two parallel, congruent point-to-image segments define the same translation. Test the claim with $A(0,0) \rightarrow A'(2,0)$ and $B(0,1) \rightarrow B'(-2,1)$. Explain the flaw and repair the geometric definition.

ANSWER _____

2

An animator rotates a rigid icon 90° counterclockwise about $O(0,0)$. Points $P(2,0)$ and $Q(0,3)$ lie on different circles centered O . Locate P' and Q' . Explain which distance is preserved for each point and what must be the same for both points.

ANSWER _____

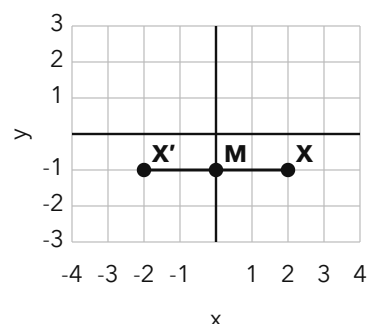
CHALLENGE

One step up

A student defines a rotation about O by only $OX' = OX$. Explain why this fails to locate a unique X' . Give two possible images of $X(2,0)$ when O is the origin, then name the extra information that selects one image.

PRACTICE 1

$X'(-2,-1)$, $M(0,-1)$. XX' is horizontal and perpendicular to the y -axis; $XM = MX' = 2$.

**PRACTICE 2**

For X off ℓ , X' is the point for which ℓ is the perpendicular bisector of XX' . For X on ℓ , $X' = X$ (fixed).

PRACTICE 3

A and B lie on the same circle centered O and the directed turn is 90° . Generally, for $X \neq O$, X' lies on the circle centered O through X with directed angle $XOX' = \theta$. O stays fixed.

PRACTICE 4

Both displacements are $(4,2)$: parallel, equal length $\sqrt{20}$, and the same direction. Each X moves to X' by $(4,2)$; equivalently XX' has the same length and direction as AB .

APPLY IT 1

The claim is false: the segments have equal length and parallel supporting lines, but opposite directions.

Their displacements are $(2,0)$ and $(-2,0)$. A translation requires the same directed displacement for every point: congruent, parallel, and the same direction (or all points fixed for zero displacement).

APPLY IT 2

$P'(0,2)$; $Q'(-3,0)$. $OP' = OP = 2$ and $OQ' = OQ = 3$; both directed turns are 90° counterclockwise. Each point stays on its own circle centered O . Different points may have different distances from O ; their rotation center and directed angle must agree.

CHALLENGE

Many points lie on the radius-2 circle. Examples (0,2) and (-2,0) both preserve distance. A specified directed angle, such as 90° counterclockwise, selects (0,2). The center alone and distance condition are insufficient.

Judge geometric definitions, including fixed points and direction, rather than accepting coordinate rules alone. Parallel translation segments may lie on the same supporting line; equal directed displacement is the precise condition. A zero translation fixes every point. Rotation angles are interpreted modulo 360° .