

Derivative Graph Detective

Use derivative graphs to sketch and explain a function.

HIGH SCHOOL • MA.912.C.3.6

MA.912.C.3.6 "Sketch graphs by using first and second derivatives. Compare the corresponding characteristics of the graphs of f , f' and f'' ."

NAME _____

DATE _____

CONNECT SHAPE AND DERIVATIVES

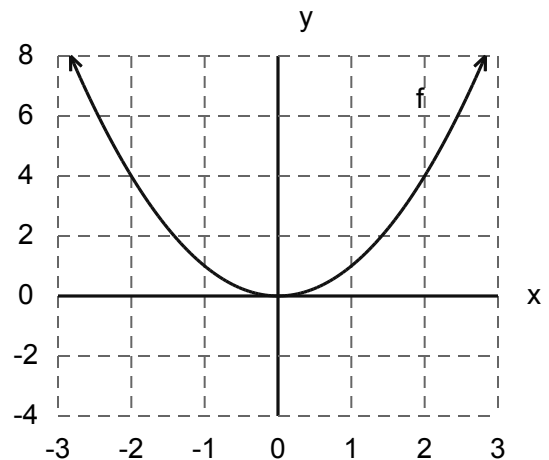
Use f' signs for increasing/decreasing behavior and extrema. Use f'' signs for concavity and inflection. A rising f' means positive f'' . Sketch f from the given information and label its features.

WORKED EXAMPLE

For the plotted $f(x)=x^2$, use $f'=2x$ and $f''=2$ to explain its shape.

1. f' changes from negative to positive at 0: a minimum.
2. f'' is always positive: concave up everywhere.

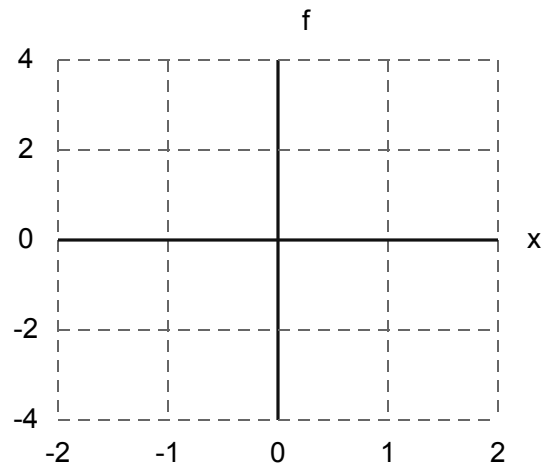
Answer: Minimum (0,0); decreases before 0, increases after; no inflection.



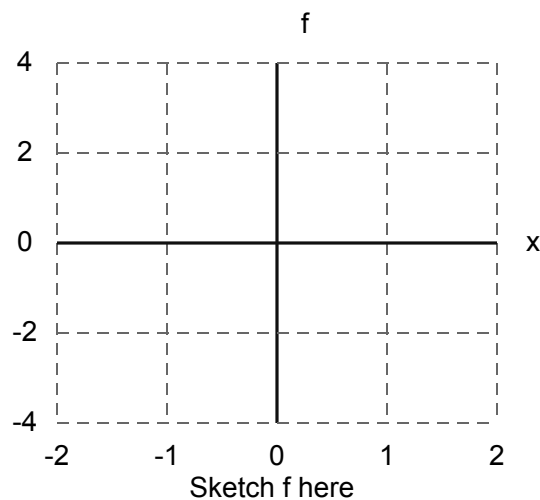
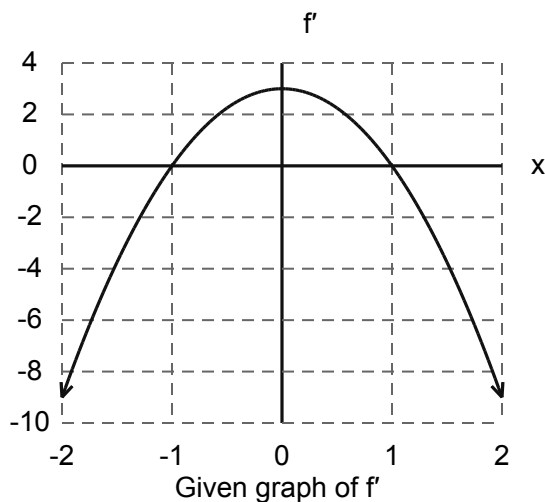
PRACTICE 4 problems

- 1 $f(x)=x^3-3x$, $f'=3x^2-3$, $f''=6x$. Sketch f ; label extrema and inflection. Explain how derivative signs determine its shape.

ANSWER _____



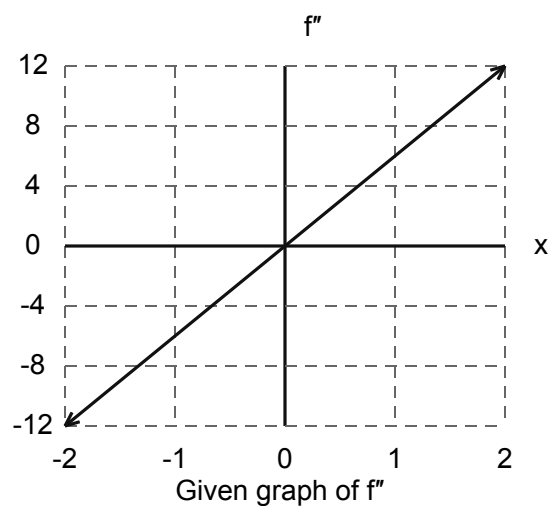
- 2** $f(x) = -x^3 + 3x$; $f' = -3x^2 + 3$ is shown and $f'' = -6x$. Sketch f . Label extrema and inflection; explain how the derivative signs reverse the previous shape.



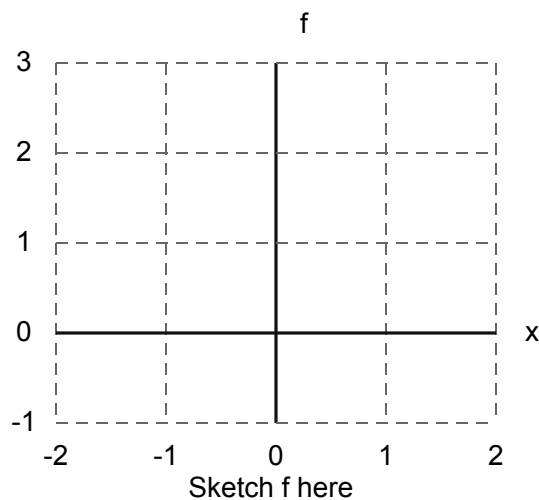
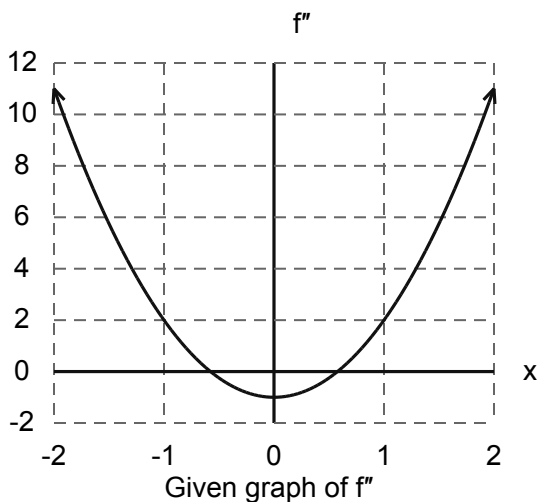
ANSWER _____

- 3** The graph shows $f'' = 6x$, where $f(x) = x^3 + 3x$ and $f' = 3x^2 + 3$. Describe how f changes and bends. Explain why there is an inflection but no extremum.

ANSWER _____



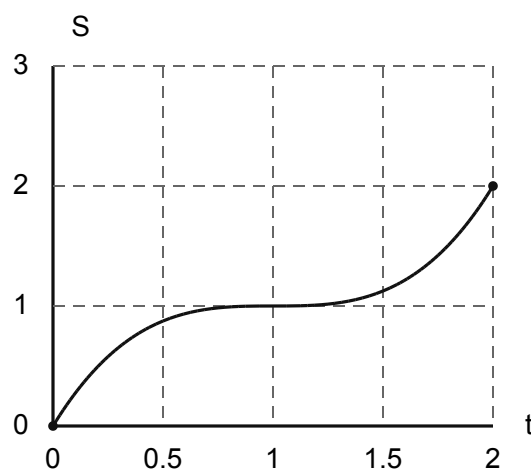
- 4 $f(x)=x^4/4-x^2/2$; $f'=x^3-x$ and the graph of $f''=3x^2-1$ are given. Sketch f ; locate all extrema and estimate the two inflection x -values.



ANSWER _____

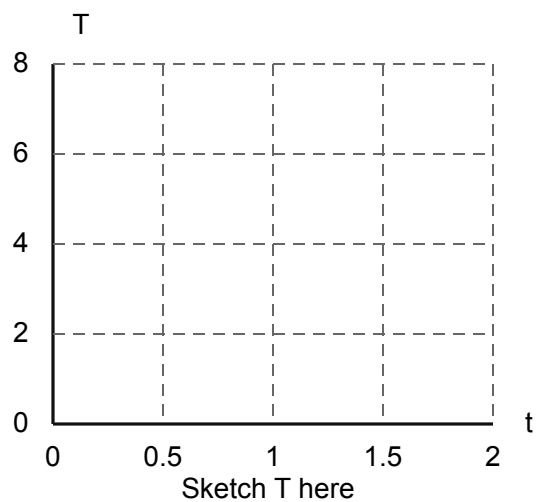
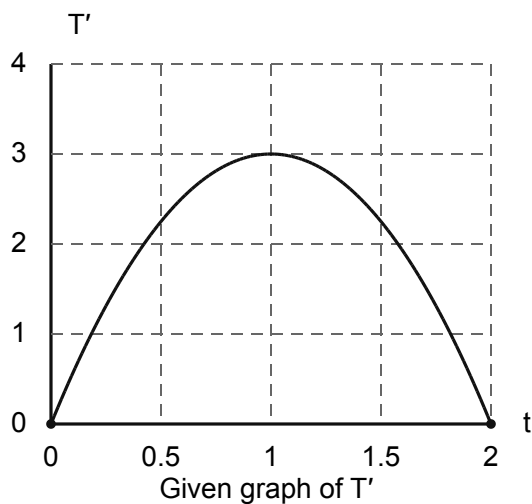
APPLY IT Show your work

- 1 A signed sensor signal $S(t)=t^3-3t^2+3t$ is graphed for $0 \leq t \leq 2$ seconds. Use $S'=3(t-1)^2$ and $S''=6t-6$ to explain its flat point and concavity. Is the flat point an extremum?



2

A device temperature is $T(t) = -t^3 + 3t^2 + 2$ degrees for $0 \leq t \leq 2$ minutes. T' is shown and $T'' = 6 - 6t$. Sketch T on the right. When is the temperature rising fastest, and how does the graph show this?

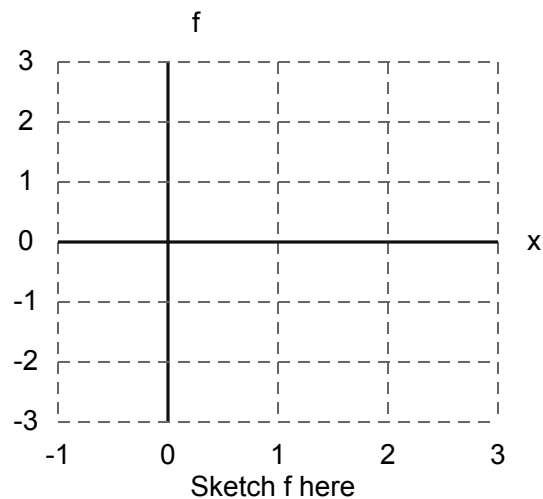


CHALLENGE One step up

Given $f'(x) = (x-1)^2$ and $f(1) = 0$, sketch f and derive f'' .

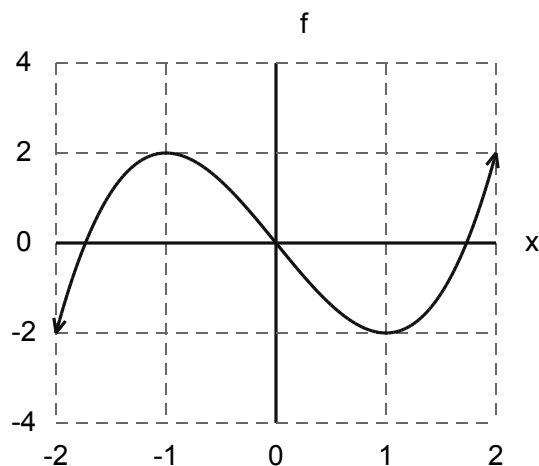
Compare features of f , f' and f'' at $x=1$ and on either side: increasing/decreasing, extrema and concavity. Does a zero of f' force an extremum of f ?

WORK SPACE AND ANSWER



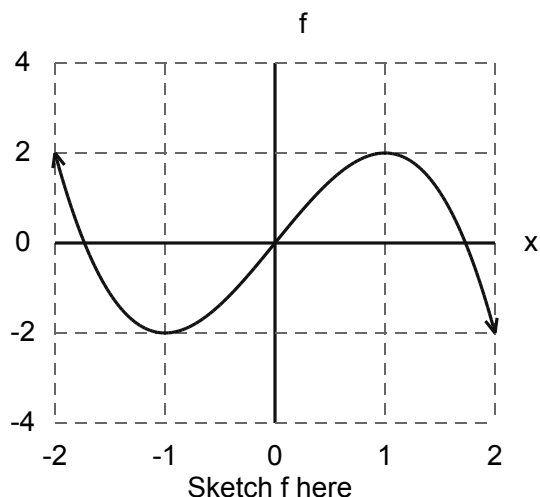
PRACTICE 1

Max $(-1,2)$, min $(1,-2)$, inflection $(0,0)$. f rises outside $[-1,1]$, falls inside; concave down left of 0, up right of 0.



PRACTICE 2

Min $(-1,-2)$, max $(1,2)$, inflection $(0,0)$. f rises on $(-1,1)$, falls outside; concave up left of 0, down right of 0.

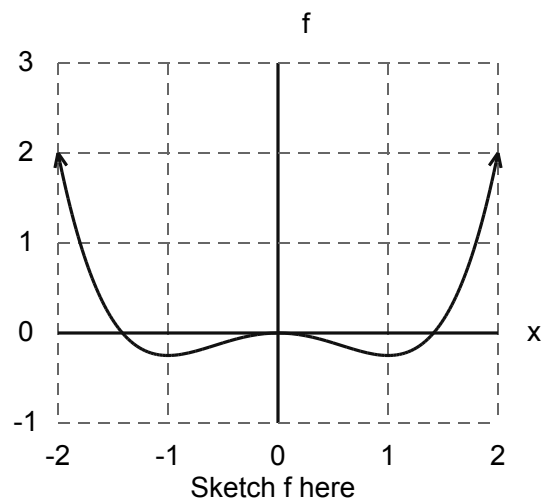


PRACTICE 3

f' is always positive: f increases with no extrema. f'' changes from negative to positive at 0; inflection $(0,0)$.

PRACTICE 4

Minima $(\pm 1, -0.25)$, max $(0,0)$; inflections at $x \approx \pm 0.577$, $y \approx -0.139$. Concave down between these x-values, up outside.



APPLY IT 1

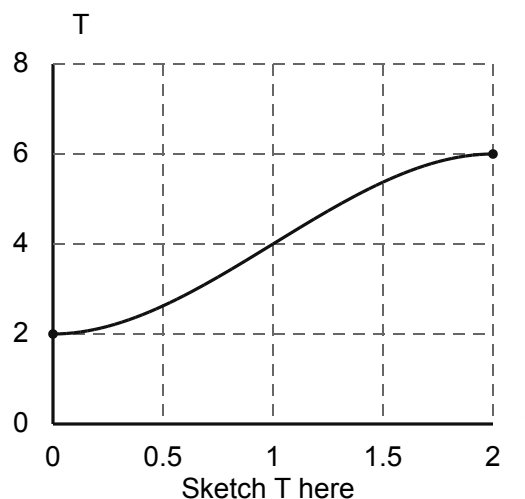
S rises throughout, with stationary inflection $(1,1)$, not an extremum. Concave down before 1, up after 1. Endpoints $(0,0),(2,2)$.

S' never changes sign; S'' changes sign at 1.

APPLY IT 2

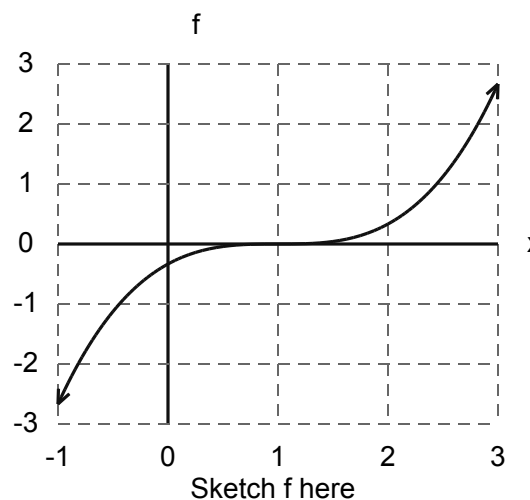
Fastest rise at $t=1$, rate 3. Inflection $(1,4)$; concave up before 1, down after. Endpoints $(0,2),(2,6)$.

A peak of T' gives the steepest slope of T .



CHALLENGE

$f=(x-1)^3/3$; $f''=2x-2$. At $x=1$: f has a stationary inflection $(1,0)$, f' has minimum $(1,0)$, and f'' crosses zero. f increases on both sides and bends down then up. f' falls then rises and is concave up. f'' rises linearly. The zero of f' does not force an extremum of f .



Require actual sketches and derivative-based explanations. Compare derivative graph signs with monotonicity and derivative graph slopes with concavity. Accept approximate inflection locations in Practice 4; exact $x=\pm 1/\sqrt{3}$, $y=-5/36$. Only interior extrema are meant; context endpoints are shown as solid dots.