

Area Tells Change

Use derivative graphs and accumulation to track function values.

HIGH SCHOOL • MA.912.C.4.5

MA.912.C.4.5 "Analyze function graphs by using derivative graphs and the Fundamental Theorem of Calculus."

NAME _____

DATE _____

SIGNED AREA CHANGES THE FUNCTION

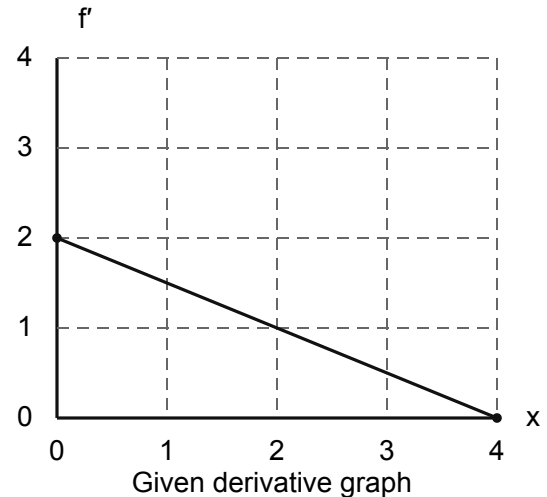
The Fundamental Theorem of Calculus gives $f(b)-f(a)$ as signed area under f' . Add areas above the axis; subtract areas below. Add the known starting f -value. Positive/negative f' means f rises/falls; rising/falling f' means concave up/down. Local extrema here mean interior points.

WORKED EXAMPLE

The graph is f' on $[0,4]$. If $f(0)=2$, find $f(4)$ and describe f on this interval.

1. Triangle area: $(1/2)(4)(2)=4$ above the axis.
2. $f(4)=2+4=6$. Positive, falling f' means f rises and is concave down.

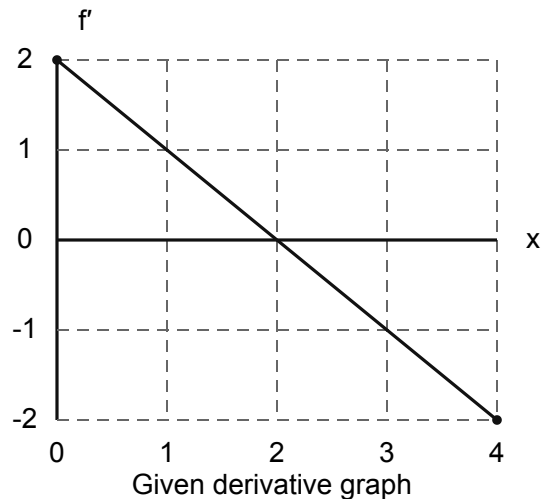
Answer: $f(4)=6$; increasing and concave down for $0 < x < 4$.



PRACTICE 4 problems

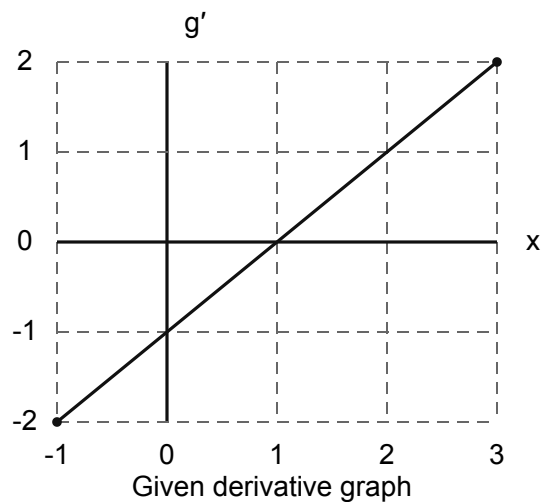
- 1 Use the graph of f' and $f(0)=1$. Find $f(4)$, the interior local extremum of f , and its increasing/decreasing intervals.

ANSWER _____



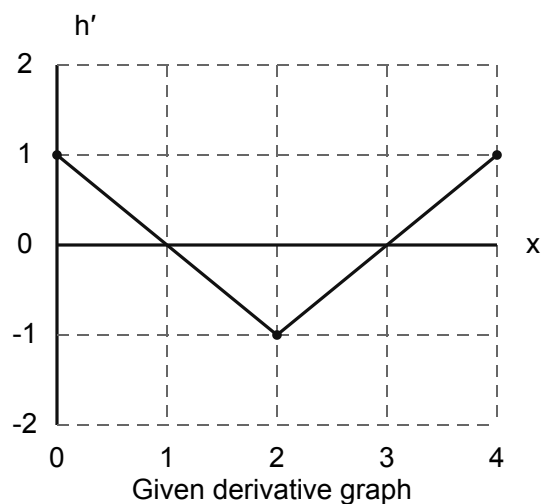
- 2 Use the graph of g' and $g(-1)=4$. Find $g(1)$, $g(3)$, the interior extremum and concavity of g .

ANSWER _____

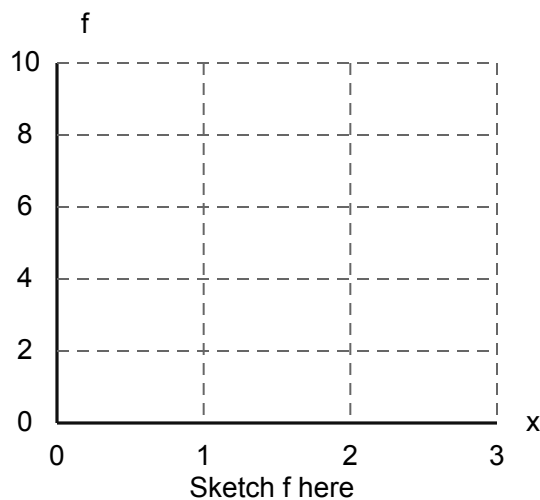
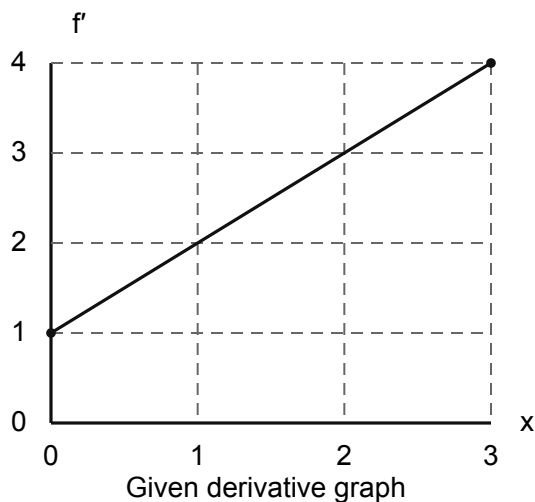


- 3 Use the graph of h' and $h(0)=2$. Find $h(4)$, all interior local extrema and the inflection point of h . Show the signed areas.

ANSWER _____



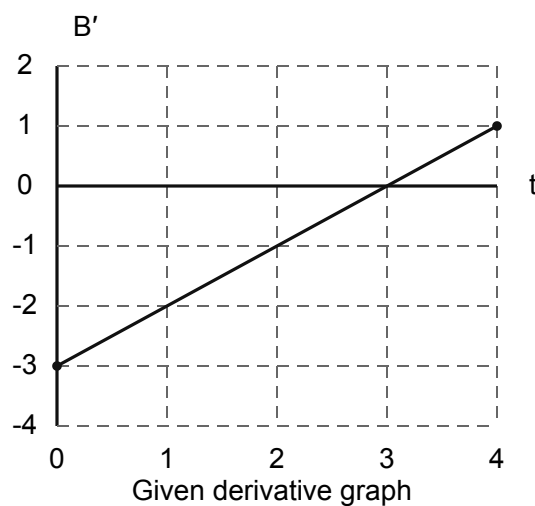
- 4** The graph shows $f'(x)=x+1$ on $[0,3]$; $f(0)=1$. Find $f(1)$, $f(2)$, $f(3)$, then sketch f on the blank grid. Describe its concavity.



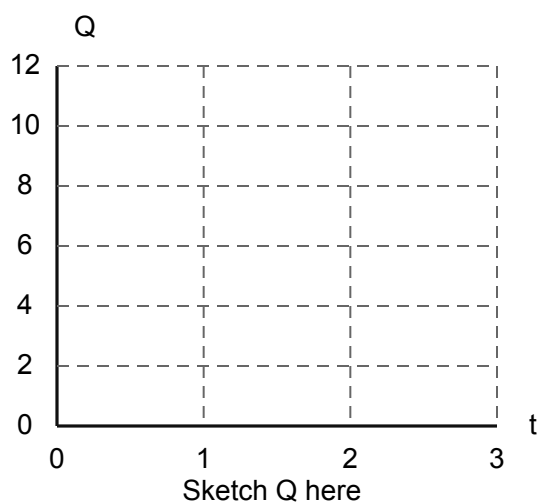
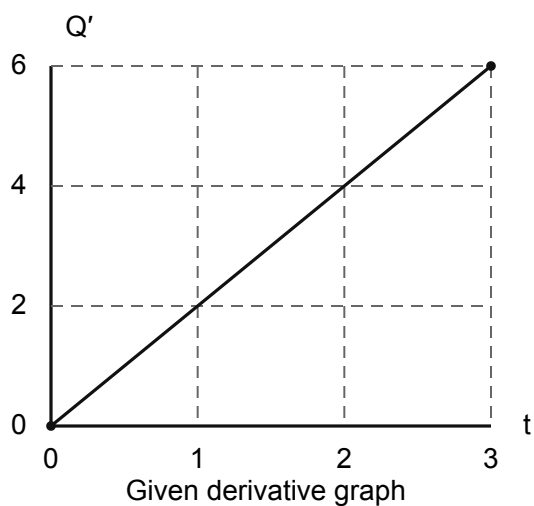
ANSWER _____

APPLY IT Show your work

- 1** A battery starts at $B(0)=70\%$ charge. The graph gives B' in percentage points per minute on $[0,4]$. Find $B(4)$ and the minimum charge, including when it occurs. Explain why the charge can end below its start even while rising near $t=4$.



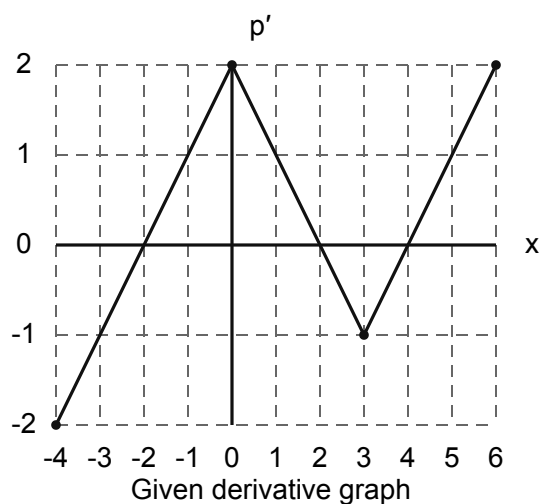
- 2** A tank initially holds $Q(0)=2$ liters. Its inflow $Q'(t)=2t$ liters/min is graphed for $0 \leq t \leq 3$. Use areas to find $Q(1)$, $Q(2)$, $Q(3)$, then sketch Q on the right.



CHALLENGE One step up

The graph gives p' ; $p(-4)=12$. Find $p(6)$, all interior local extrema (coordinates), and the absolute maximum and minimum of p on $[-4,6]$. Include endpoint comparisons.

WORK SPACE AND ANSWER



PRACTICE 1

$f(4)=1$; local maximum $(2,3)$. Increasing $(0,2)$, decreasing $(2,4)$.

PRACTICE 2

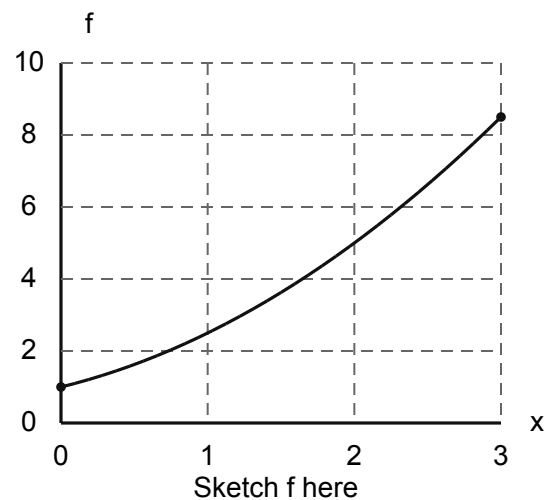
$g(1)=4-(1/2)(2)(2)=2$; $g(3)=2+(1/2)(2)(2)=4$. Minimum $(1,2)$; concave up on $(-1,3)$ because g' rises.

PRACTICE 3

$h(4)=2$; max $(1,2.5)$, min $(3,1.5)$, inflection $(2,2)$. Signed areas $+0.5,-0.5,-0.5,+0.5$.

PRACTICE 4

$f(1)=2.5$, $f(2)=5$, $f(3)=8.5$. $f=1+x+x^2/2$; increasing and concave up.

**APPLY IT 1**

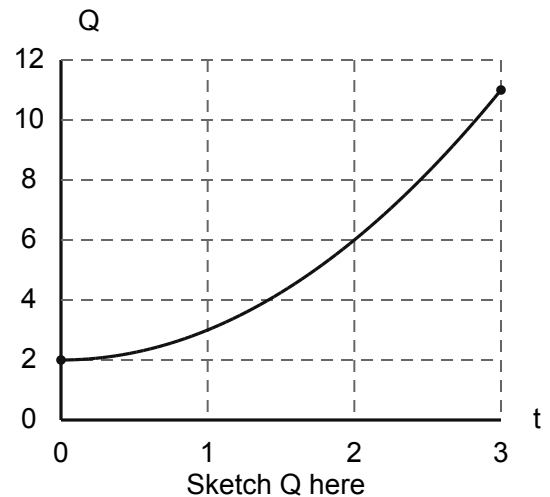
$B(4)=66\%$; minimum 65.5% at $t=3$. Signed areas -4.5 and $+0.5$ give net -4 points.

Negative area before $t=3$ exceeds positive area afterward; a positive final rate does not undo earlier losses.

APPLY IT 2

$Q(t)=2+t^2$; $Q(1)=3$, $Q(2)=6$, $Q(3)=11$ liters. Increasing, concave up.

Each area from 0 to t is a triangle of area t^2 ; add the initial 2 liters.

**CHALLENGE**

Signed areas: $[-4,-2]$: -2; $[-2,0]$: +2; $[0,2]$: +2; $[2,3]$: -0.5; $[3,4]$: -0.5; $[4,6]$: +2. Thus $p(-2)=10$, $p(0)=12$, $p(2)=14$, $p(3)=13.5$, $p(4)=13$, $p(6)=15$. Local minima $(-2,10)$, $(4,13)$; local max $(2,14)$. Absolute min 10 at -2; absolute max 15 at 6.

Use signed triangular/trapezoidal areas, not total unsigned area. Accept equivalent antiderivative calculations with the correct starting constant. Require sketches in Practice 4 and Apply 2. Distinguish a maximum of the derivative (steepest slope) from a maximum of the function.