

# Residual Reality Check

*Use residuals to judge a linear model.*

**HIGH SCHOOL • MA.912.DP.2.5**

**MA.912.DP.2.5** "Given a scatter plot that represents bivariate numerical data, assess the fit of a given linear function by plotting and analyzing residuals."

NAME \_\_\_\_\_

DATE \_\_\_\_\_

## KEEP BOTH COORDINATES

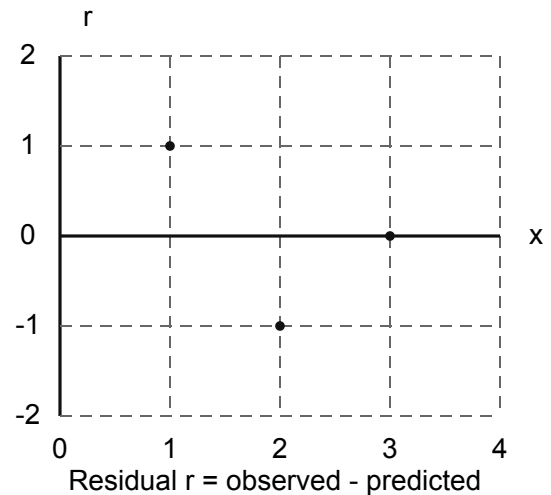
Residual  $r = \text{observed } y - \text{predicted } y$ . Plot  $(x, r)$  as separate points. Positive is above the line; negative is below. Assess residual size relative to data variation and look for trends or curves. A bad given line may still allow a better linear model.

### WORKED EXAMPLE

For model  $y=3x+2$ , observed points are  $(1,6),(2,7),(3,11)$ . Compute and plot the residuals.

1. Predictions are 5,8,11; subtract them from 6,7,11.
2. The residuals are 1,-1,0. Plot  $(1,1),(2,-1),(3,0)$ , without joining the dots.
3. Positive is above the model; negative is below; zero is exactly on it.

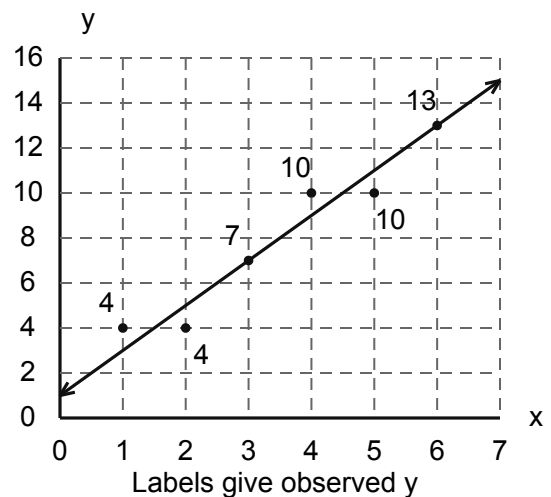
**Answer: Residual points  $(1,1),(2,-1),(3,0)$ .**



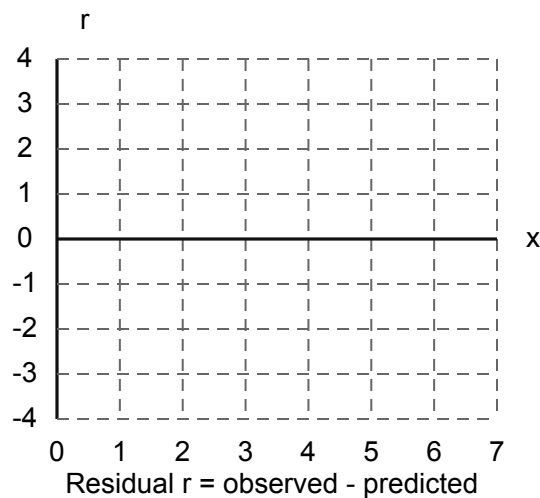
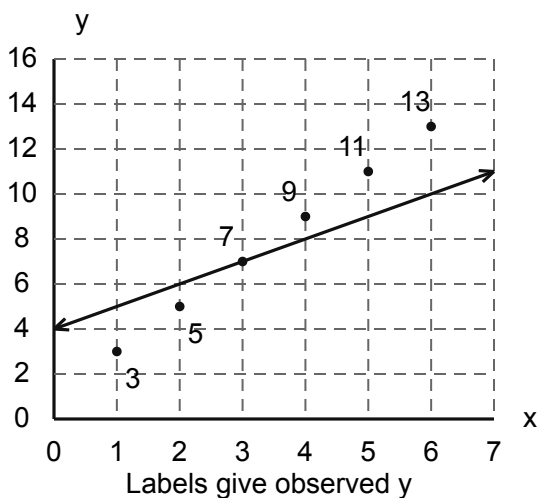
## PRACTICE 4 problems

- 1** The line is  $y=2x+1$ . Read the plotted point at  $x=4$  and find its residual. Explain its sign. Then count positive, negative and zero residuals for all six points.

**ANSWER** \_\_\_\_\_  
 \_\_\_\_\_



- 2** Explain the residual trend. Is this particular line suitable? Can a different line fit these data exactly?  
Model:  $y=1x+4$ . Plot residuals against  $x$  on the blank grid.



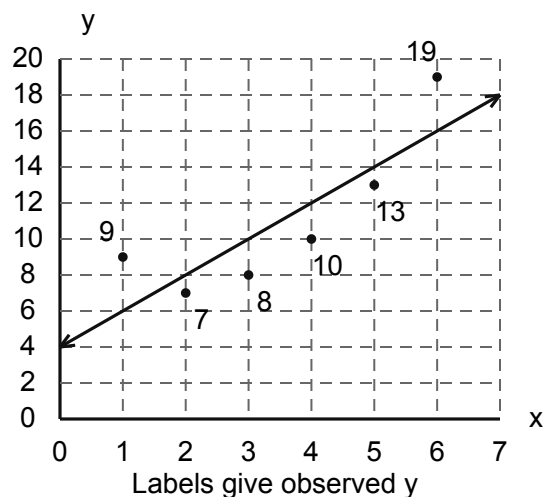
ANSWER \_\_\_\_\_

\_\_\_\_\_

- 3** The scatterplot has line  $y=2x+4$ . Its residuals, in  $x$  order, are 3, -1, -2, -2, -1, 3. Describe the pattern and assess the line. Does the evidence suggest trying a nonlinear model?

ANSWER \_\_\_\_\_

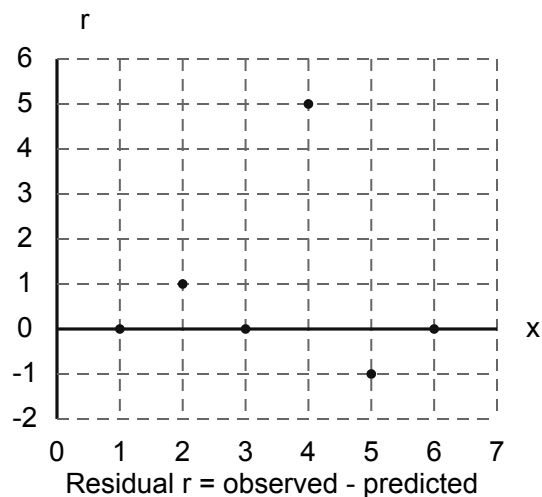
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- 4** These residuals come from  $y=x+2$ . Find the largest absolute residual and recover its observed point. Count positive, negative and zero residuals. Explain why sign counts alone are inadequate.

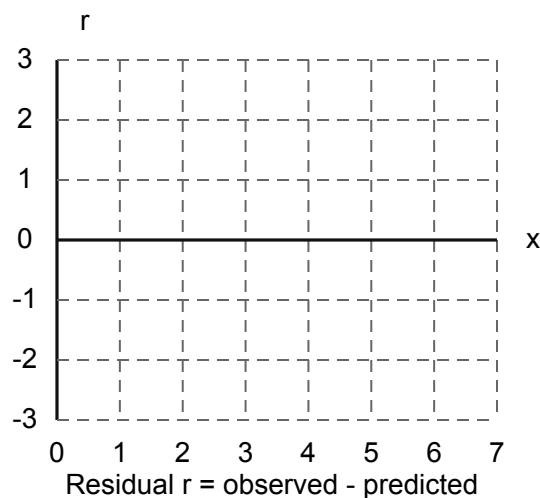
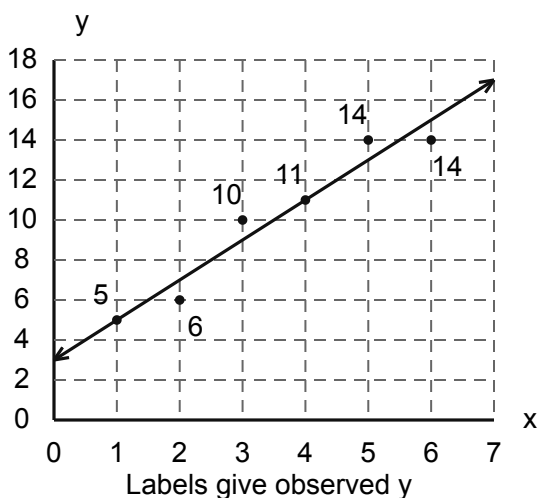
**ANSWER** \_\_\_\_\_

\_\_\_\_\_



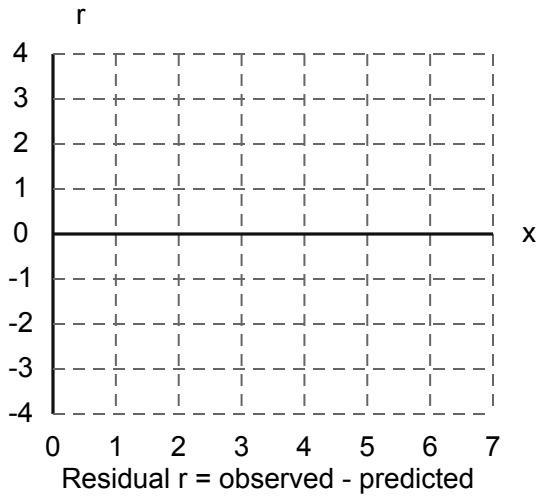
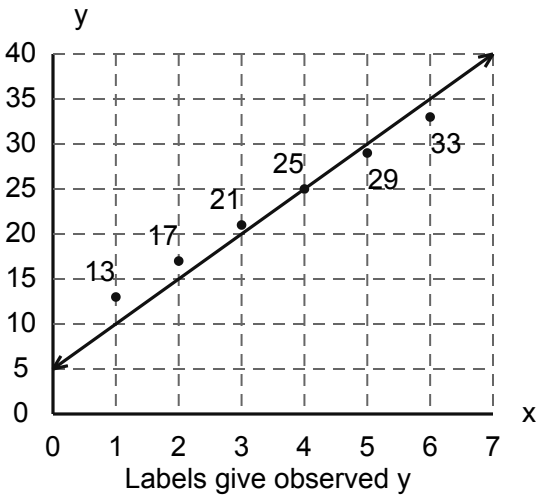
## APPLY IT Show your work

- 1** A class records daily light  $x$  (hours) and seedling height  $y$  (cm). Assess this line; explain the sign at  $x=3$  in context. Model:  $y=2x+3$ . Plot residuals against  $x$  on the blank grid.



2

A coach records ride time  $x$  (hours) and distance  $y$  (km). Assess the given line. What slope correction does the residual pattern suggest? Model:  $y=5x+5$ . Plot residuals against  $x$  on the blank grid.



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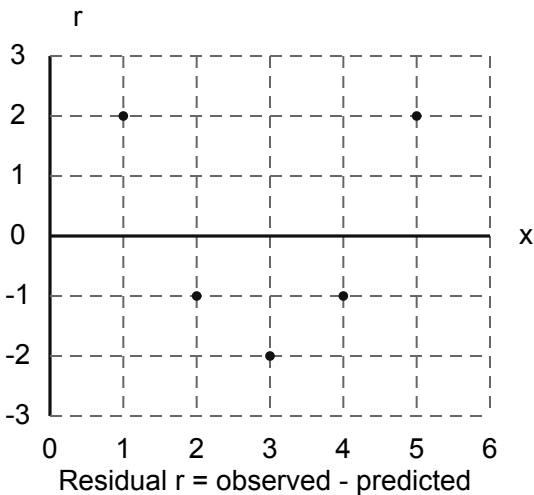
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**CHALLENGE** One step up

These plotted residuals come from  $y=2x+4$ . A student says, "Residuals sum to zero, so the model must fit well." Test the claim using both the sum and the pattern.

WORK SPACE AND ANSWER

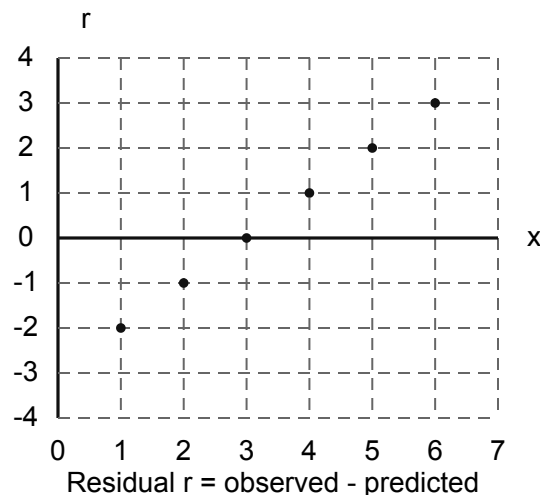


## PRACTICE 1

At  $x=4$ , observed  $y=10$ , predicted  $y=9$ :  $r=1$ , above the line. Two positive, two negative, two zero residuals.

## PRACTICE 2

Residuals  $-2,-1,0,1,2,3$  rise systematically. Given slope is too small;  $y=2x+1$  fits all points exactly.



## PRACTICE 3

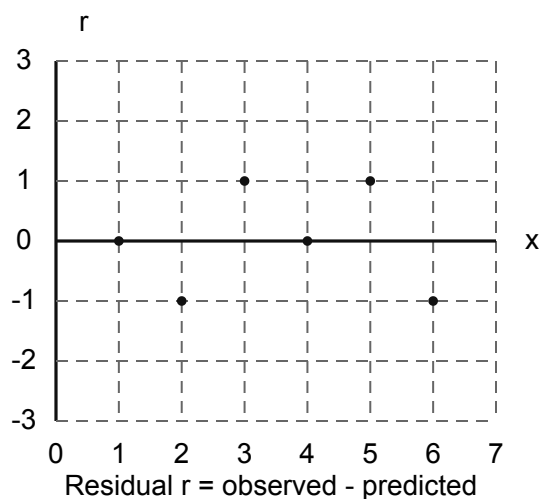
Residuals  $3,-1,-2,-2,-1,3$  form a U-shape. The line misses systematic curvature; try a nonlinear model.

## PRACTICE 4

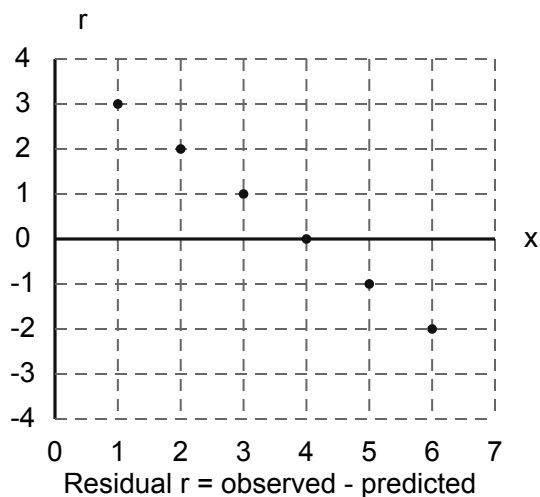
Residuals  $0,1,0,5,-1,0$ . Largest  $|r|=5$  at  $(4,11)$ ; two positive, one negative, three zero. The outlier is hidden by sign counts alone.

**APPLY IT 1**

Residuals 0,-1,1,0,1,-1: small without a trend/curve. At  $x=3$ , height is 1 cm above prediction; reasonable fit.  
Subtract predicted height from observed height (cm).

**APPLY IT 2**

Residuals 3,2,1,0,-1,-2. Reduce slope by 1:  
 $y=4x+9$  fits every point.  
Falling residuals indicate the given slope is too large.

**CHALLENGE**

**Residuals 2,-1,-2,-1,2 sum to 0 but form a U-shape. Cancellation does not establish a good linear fit; the curvature needs attention.**

Require separate residual points plotted at their correct  $x$ -values, with no connecting segments. A reasonable fit is an informal judgment from size and pattern, not a proof from a small sample. Upward/downward residual trends diagnose this given line; they do not rule out a different linear fit.