

Probability Pairings

Use two-way tables to test independence.

HIGH SCHOOL • MA.912.DP.4.5

MA.912.DP.4.5 "Given a two-way table containing data from a population, interpret the joint and marginal relative frequencies as empirical probabilities and the conditional relative frequencies as empirical conditional probabilities..."

NAME _____

DATE _____

CHOOSE THE RIGHT DENOMINATOR

Each table describes the whole stated group; choose one member at random. Joint probability uses a cell divided by the grand total. Marginal probability uses a row or column total divided by the grand total. Conditional probability uses only the given subgroup. Exact independence requires equal conditional and marginal probabilities. For approximate independence in this worksheet, use a difference of at most 3 percentage points; this is an exercise criterion, not a universal rule.

WORKED EXAMPLE

Find $P(\text{podcasts})$ and $P(\text{podcasts} \mid \text{in debate})$. Are the characteristics exactly independent? Approximately independent under our criterion?

ALL 70 STUDENTS IN A CLUB PROGRAM

Debate status	Podcasts	No podcasts	Total
In debate	13	17	30
Not in debate	22	18	40
Total	35	35	70

1. Marginal: $P(\text{podcasts}) = 35/70 = 50\%$.
2. Conditional: $P(\text{podcasts} \mid \text{in debate}) = 13/30 \approx 43.33\%$.
3. The difference is about 6.67 percentage points, greater than 3.

Answer: Neither exactly nor approximately independent by the stated criterion.

PRACTICE 4 problems

- 1** Use the whole group shown. Find $P(\text{in art and bikes})$. Explain why the denominator is the grand total, not the art-club total.

ALL 46 STUDENTS IN AN ACTIVITY GROUP

Art status	Bikes	Walks	Total
In art	6	11	17
Not in art	14	15	29
Total	20	26	46

ANSWER _____

- 3** Find $P(\text{pet} \mid \text{takes drama})$ and $P(\text{pet})$. Are these characteristics exactly independent? Show both denominators.

ALL 36 STUDENTS IN A DRAMA SURVEY GROUP

Drama status	Pet	No pet	Total
Takes drama	4	8	12
No drama	8	16	24
Total	12	24	36

ANSWER _____

- 2** Find $P(\text{games} \mid \text{in band})$ and $P(\text{games})$. Decide exact independence and approximate independence using the stated 3-point criterion.

ALL 36 STUDENTS IN A MUSIC PROGRAM

Band status	Games	No games	Total
In band	5	7	12
Not in band	11	13	24
Total	16	20	36

ANSWER _____

- 4** Find $P(\text{permit})$, then $P(\text{in auto and permit})$. Label each result marginal or joint.

ALL 25 STUDENTS IN A DRIVER PROGRAM

Auto status	Permit	No permit	Total
In auto	6	4	10
Not in auto	9	6	15
Total	15	10	25

ANSWER _____

APPLY IT Show your work

- 1** For this entire lunch group, find $P(\text{packs} \mid \text{bottle})$ and $P(\text{packs})$. Interpret each denominator and assess approximate independence under the stated criterion.

ALL 80 STUDENTS IN THE LUNCH GROUP

Lunch	Bottle	No bottle	Total
Packs	12	28	40
Buys	30	10	40
Total	42	38	80

ANSWER _____

- 2** Find $P(\text{esports and coding})$, $P(\text{esports} \mid \text{coding})$, and $P(\text{esports})$. Assess exact independence and interpret the joint probability in this group.

ALL 50 STUDENTS IN A TECHNOLOGY GROUP

Coding	Esports	No esports	Total
Takes coding	18	12	30
No coding	12	8	20
Total	30	20	50

ANSWER _____

CHALLENGE One step up

Compare $P(\text{job} \mid \text{sport})$ with $P(\text{job})$. Are the characteristics exactly independent? Approximately independent under our 3-point criterion? Explain why unequal probabilities do not automatically defeat the approximate judgment.

ALL 200 JUNIORS AT A SCHOOL

Sports	Paid job	No paid job	Total
Plays sport	31	48	79
No sport	47	74	121
Total	78	122	200

WORK SPACE AND ANSWER

PRACTICE 1

Use the whole group shown. Find $P(\text{in art and bikes})$. Explain why the denominator is the grand total, not the art-club total.

$6/46=3/23\approx 13.04\%$. This joint event is measured out of all 46 students.

PRACTICE 2

Find $P(\text{games} \mid \text{in band})$ and $P(\text{games})$. Decide exact independence and approximate independence using the stated 3-point criterion.

$5/12\approx 41.67\%$ and $16/36\approx 44.44\%$. Not exactly independent; approximately independent (difference about 2.78 points).

PRACTICE 3

Find $P(\text{pet} \mid \text{takes drama})$ and $P(\text{pet})$. Are these characteristics exactly independent? Show both denominators.

$4/12=1/3$ and $12/36=1/3$. Exactly independent; also approximately independent.

PRACTICE 4

Find $P(\text{permit})$, then $P(\text{in auto and permit})$. Label each result marginal or joint.

$P(\text{permit})=15/25=3/5$ (marginal). $P(\text{in auto and permit})=6/25$ (joint).

APPLY IT 1

$12/42=2/7\approx 28.57\%$ versus $40/80=50\%$. Difference about 21.43 points: not approximately independent.

Conditioning on bottle restricts the group to 42 students; the marginal probability uses all 80.

APPLY IT 2

$18/50=36\%$ like esports and take coding. Conditional $18/30=60\%$; marginal $30/50=60\%$. Exactly independent.

Use the 30 coding students for the conditional probability and all 50 students for the joint and marginal probabilities.

CHALLENGE

$31/79\approx 39.24\%$; $78/200=39\%$. Not exactly independent; approximately independent: difference is about 0.24 percentage points.

Accept fractions or sensible percentage rounding. Percentage-point difference is the absolute difference of percentages, not a relative percentage change. The 3-point cutoff applies only to these exercises. Equal probabilities show exact independence within the stated finite population; approximate equality uses the supplied criterion.