

Slice and Classify

Start with the conic; explain the plane that produces it.

HIGH SCHOOL • MA.912.GR.7.1

MA.912.GR.7.1 "Given a conic section, describe how it can result from the slicing of two cones."

NAME _____

DATE _____

A CONIC COMES FROM A CUT

The two nappes meet at V and extend forever. Away from V: a horizontal cut gives a circle; a tilted cut across one nappe, less steep than a generator, gives an ellipse; a cut parallel to a generator gives a parabola; cutting both nappes gives a hyperbola. A cut through V can be degenerate. The side view shows a plane edge-on.

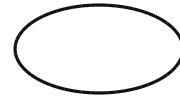
WORKED EXAMPLE

The given section is an ellipse. Describe the shown plane and explain why the cut is closed.

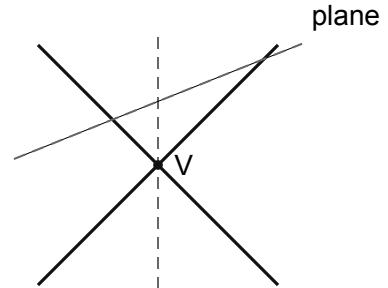
1. The plane misses V and crosses only one nappe.
2. It is tilted but less steep than a generating line.
3. The intersection closes around the cone.

Answer: An oblique one-nappe cut, not parallel to a generator.

GIVEN: ELLIPSE



AXIAL SIDE VIEW

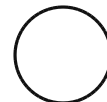


Cones extend beyond this view.

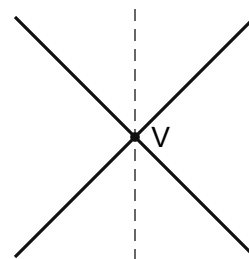
PRACTICE 4 problems

- 1** Given a circle, draw one cutting-plane line in the side view. Describe its relation to the axis and whether it passes through V.

GIVEN: CIRCLE



AXIAL SIDE VIEW



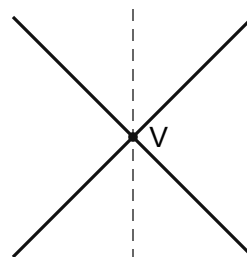
Cones extend beyond this view.

- 2** Given a parabola, draw a possible plane edge-on.
State which line it is parallel to and why the plane must miss V.

GIVEN: PARABOLA



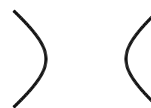
AXIAL SIDE VIEW



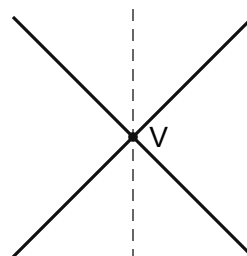
Cones extend beyond this view.

- 3** Given a hyperbola, draw a possible cutting plane.
Explain why there are two branches.

GIVEN: HYPERBOLA



AXIAL SIDE VIEW



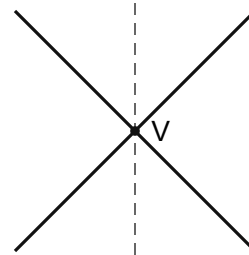
Cones extend beyond this view.

- 4** The section is a single point. Draw a plane that produces it and identify the point.

GIVEN: POINT



AXIAL SIDE VIEW



Cones extend beyond this view.

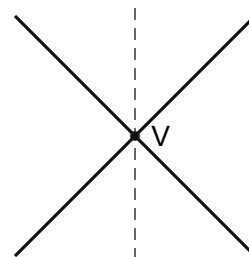
APPLY IT Show your work

- 1** A design app must produce a parabola from a double cone. Its current plane is perpendicular to the axis and misses V. Explain how to rotate and position the plane to get the target. Draw the final cut.

GIVEN: PARABOLA



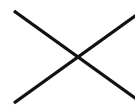
AXIAL SIDE VIEW



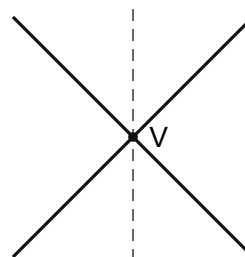
Cones extend beyond this view.

- 2** A modeler wants two intersecting straight lines instead of a curved hyperbola. Describe and draw a plane that gives this degenerate section.

GIVEN: TWO LINES



AXIAL SIDE VIEW



Cones extend beyond this view.

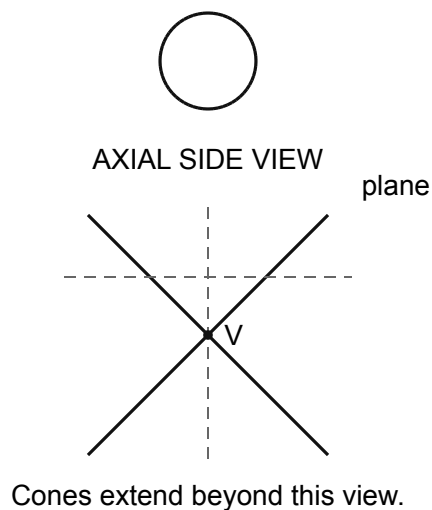
CHALLENGE One step up

A tilted plane misses V and intersects just one nappe. Is that enough to guarantee an ellipse? Give the missing condition and a counterexample.

PRACTICE 1

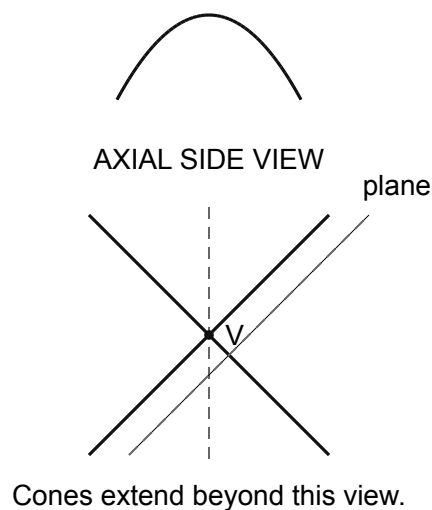
A plane perpendicular to the axis, missing V, cuts one nappe in a circle.

GIVEN: CIRCLE

**PRACTICE 2**

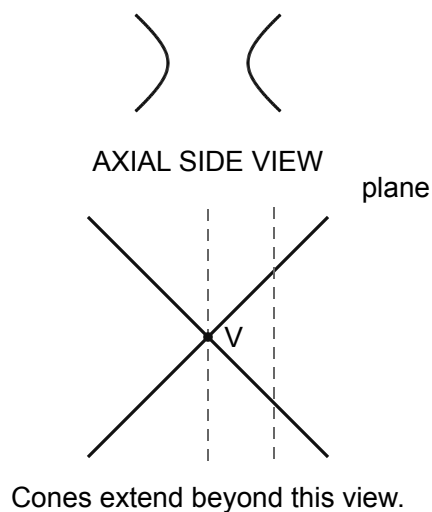
Parallel to a generating line and missing V. Passing through V instead can produce a degenerate line.

GIVEN: PARABOLA

**PRACTICE 3**

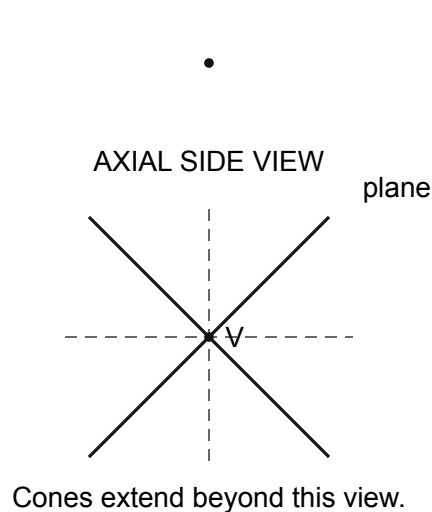
A plane cuts both nappes and misses V. Each nappe supplies one branch.

GIVEN: HYPERBOLA

**PRACTICE 4**

A plane through V perpendicular to the axis meets the cone only at V.

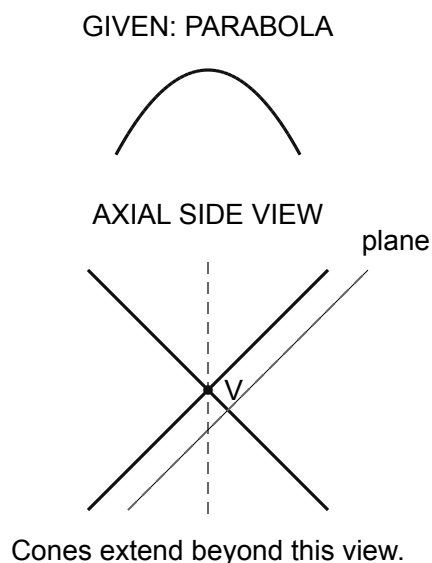
GIVEN: POINT



APPLY IT 1

Tilt the plane until parallel to a generator while keeping it away from V.

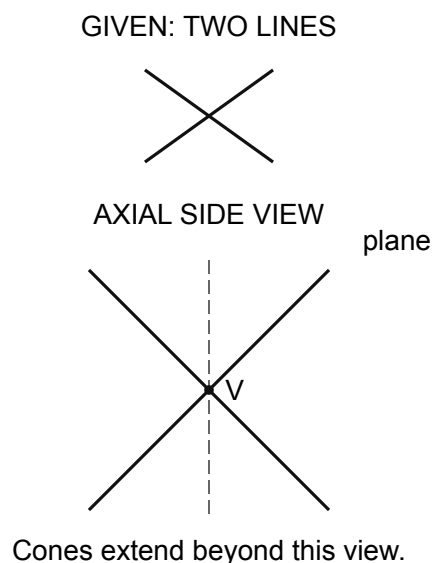
A horizontal cut gives a circle. A non-tip cut parallel to a generating line gives the requested parabola.



APPLY IT 2

Use a plane containing the axis, so it passes through V.

The plane contains two generating lines, which intersect at V. Passing through the shared vertex is the essential exception.



CHALLENGE

No. The tilted plane must be less steep than a generator. A plane parallel to a generator can cut one nappe in a parabola instead, even though it misses V.

Accept rotated or translated valid cuts. Dashed line placement is illustrative; slope/orientation and avoiding or including V are the essential relationships. Students may label their plane line.