

Hyperbola Path Finder

Graph both branches and interpret their features.

HIGH SCHOOL • MA.912.GR.7.9

MA.912.GR.7.9 "Graph and solve mathematical and real-world problems that are modeled with an equation of a hyperbola. Determine and interpret key features in terms of the context."

NAME _____

DATE _____

READ THE POSITIVE TERM

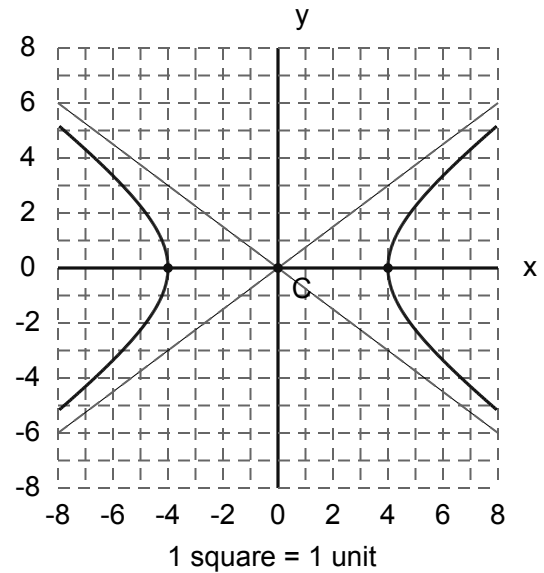
For $(x-h)^2/a^2 - (y-k)^2/b^2 = 1$, vertices are $(h \pm a, k)$; asymptotes have slopes $\pm b/a$. For $(y-k)^2/a^2 - (x-h)^2/b^2 = 1$, vertices are $(h, k \pm a)$; slopes are $\pm a/b$. In both, $c^2 = a^2 + b^2$, eccentricity $e = c/a$, and foci lie c units from the center on the transverse axis. Directrices are a^2/c units from the center. Use the supplied grids.

WORKED EXAMPLE

Graph $x^2/16 - y^2/9 = 1$. Find its center, vertices and asymptotes.

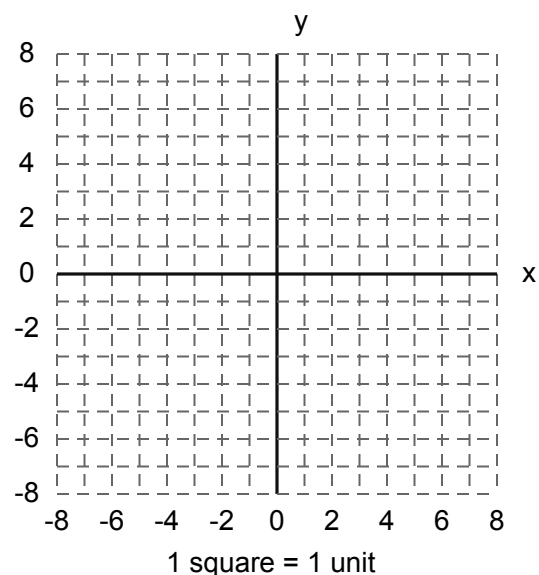
1. $C = (0,0)$; $a = 4$, $b = 3$. The x -term is positive.
2. Vertices: $(-4,0)$, $(4,0)$. Asymptotes: $y = \pm(3/4)x$.
3. Sketch two branches through the vertices approaching these lines.

Answer: The shown graph opens left and right.

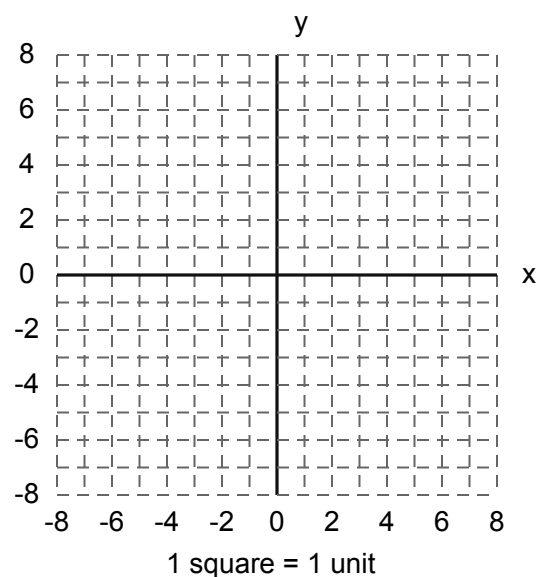


PRACTICE 4 problems

- 1** On this grid, graph $(x-1)^2/9 - (y+1)^2/16 = 1$. Mark center and vertices; draw asymptotes. State domain and range.



- 2** Graph $(y-1)^2/9 - (x+1)^2/16 = 1$ on this grid. Give center, vertices, asymptotes, domain and range.

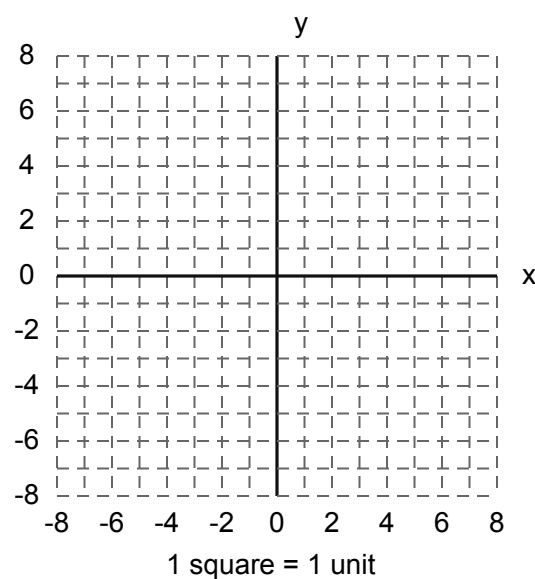


- 3** For $x^2/9 - y^2/16 = 1$, find foci, eccentricity, directrices, and transverse/conjugate axis lengths.

- 4** Solve the system $x^2/9 - y^2/16 = 1$ and $y=4$. Give exact coordinates and explain why there are two solutions.

APPLY IT Show your work

- 1** A location model has transmitters at $(-5,0)$ and $(5,0)$, in km. Points with absolute distance difference 6 km satisfy $x^2/9 - y^2/16 = 1$. Graph the locus here. Find possible positions when $y=0$ and explain what the two foci represent.



2

A tower cross-section is modeled by $x^2/9 - y^2/16 = 1$, with y the vertical distance from the narrowest level and x the signed horizontal distance from its centerline, in metres. At $y=0$ find its width; at $y=4$ find its width exactly. Explain the vertices.

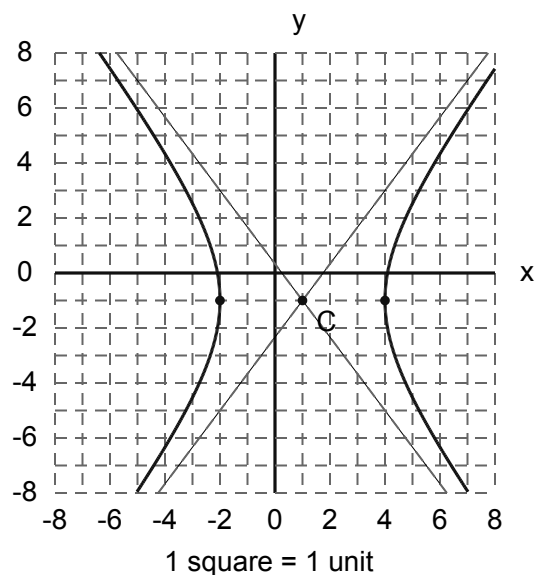
CHALLENGE One step up

For $(y-1)^2/9 - (x+1)^2/16 = 1$, give foci, directrices, eccentricity, and both axis lengths. A student uses slopes $\pm 4/3$ for its asymptotes. Explain the error.

PRACTICE 1

$C=(1,-1)$; $V=(-2,-1),(4,-1)$; $y+1=\pm(4/3)(x-1)$.

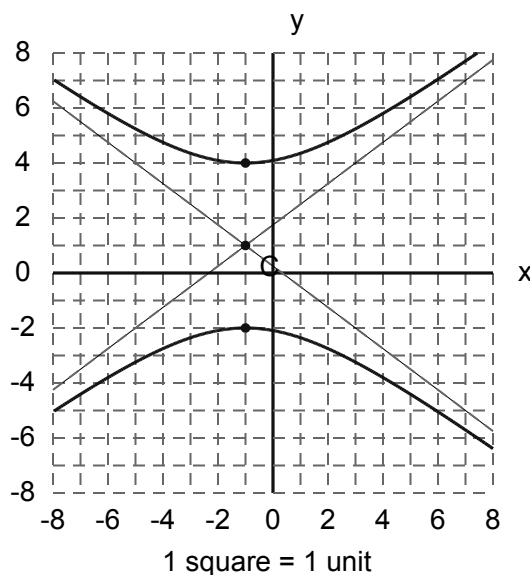
Domain $(-\infty,-2] \cup [4,\infty)$; range all real numbers.



PRACTICE 2

$C=(-1,1)$; $V=(-1,-2),(-1,4)$; $y-1=\pm(3/4)(x+1)$.

Domain all real numbers; range $(-\infty,-2] \cup [4,\infty)$.



PRACTICE 3

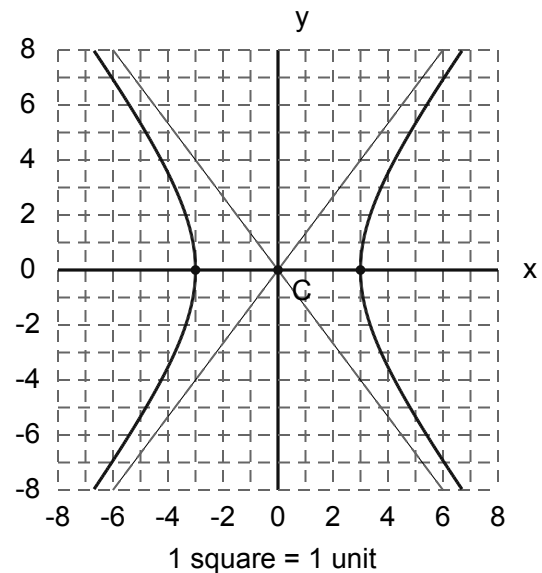
$c=5$; foci $(\pm 5,0)$; $e=5/3$; directrices $x=\pm 9/5$; transverse axis 6, conjugate axis 8.

PRACTICE 4

$(\pm 3\sqrt{2},4)$. Substitution gives $x^2/9=2$; both left and right branches meet $y=4$.

APPLY IT 1

Positions $(-3,0)$ and $(3,0)$; foci are the transmitter locations.
At $y=0$, $x^2/9=1$ gives $x=\pm 3$. Each point is 2 km from one transmitter and 8 km from the other, a difference of 6 km.



APPLY IT 2

Narrowest width 6 m; at $y=4$ width $6\sqrt{2}$ m. Vertices $(\pm 3,0)$ locate the two closest side points.
At $y=0$ the sides are $x=\pm 3$. At $y=4$ they are $x=\pm 3\sqrt{2}$; width is right x minus left x .

CHALLENGE

Foci $(-1,6), (-1,-4)$; directrices $y=1\pm 9/5$; $e=5/3$; transverse axis 6, conjugate axis 8. The y -term is positive, so the asymptote slopes are $\pm a/b = \pm 3/4$.

Accept equivalent interval, inequality or set-builder notation. The foci lie on the transverse axis; directrices are perpendicular to it. Curves extend beyond the printed window.