

Shortest Route Lab

Read weighted graphs and prove a route is cheapest.

HIGH SCHOOL • MA.912.LT.2

MA.912.LT.2 "Apply optimization and techniques from Graph Theory to solve problems."

NAME _____

DATE _____

COST IS ON THE EDGE

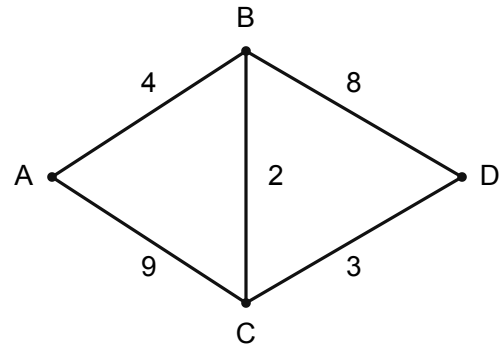
All graphs here are undirected with positive weights. Add edge weights along a route. Compare every simple route to prove a shortest path; a detour cycle only adds cost. A spanning tree instead connects every vertex with no cycle. These are different optimization tasks.

WORKED EXAMPLE

Find the shortest route from A to D. Compare all simple routes.

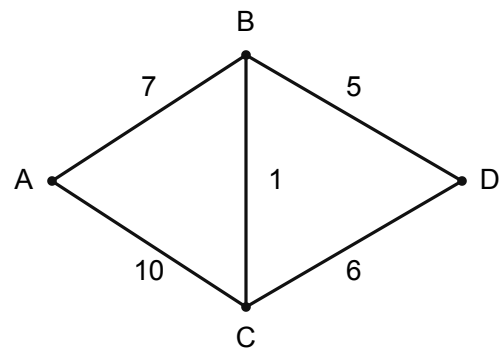
1. A-B-D: 12; A-C-D: 12.
2. A-B-C-D: 9; A-C-B-D: 19.
3. All four simple routes are checked.

Answer: A-B-C-D, weight 9.

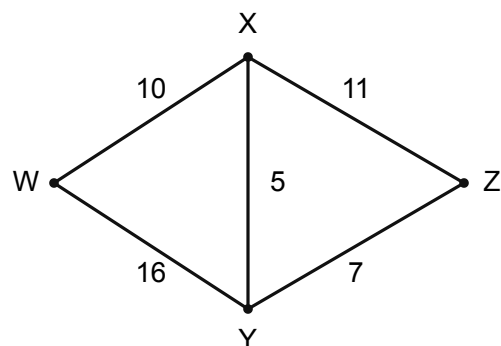


PRACTICE 4 problems

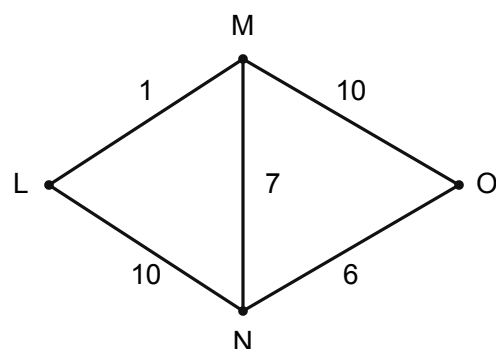
- 1** Find the shortest route from A to D. List all four simple routes and their totals.



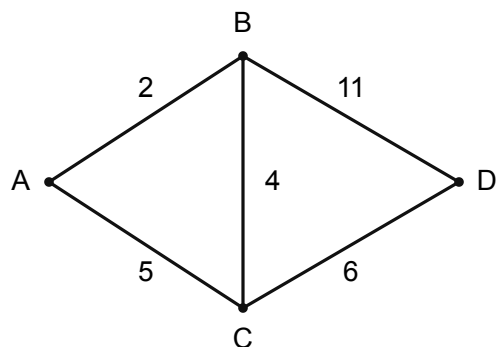
- 2** Find the shortest route from W to Z. List all four simple routes and their totals.



- 3** Find the shortest route from L to O. List all four simple routes and their totals.

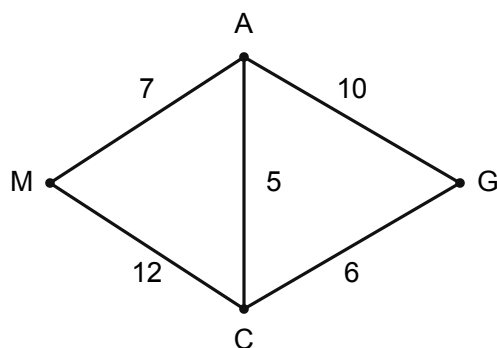


- 4** A designer chooses edge AB first because it is cheapest from A. Does this lead to a shortest A-to-D route? Compare all routes.

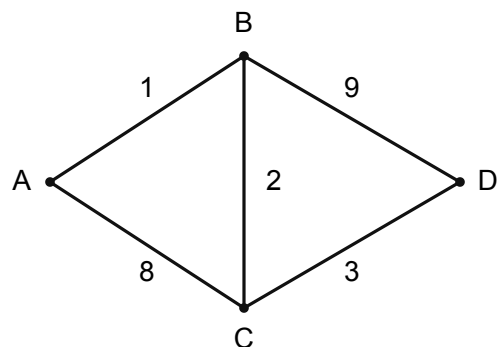


APPLY IT Show your work

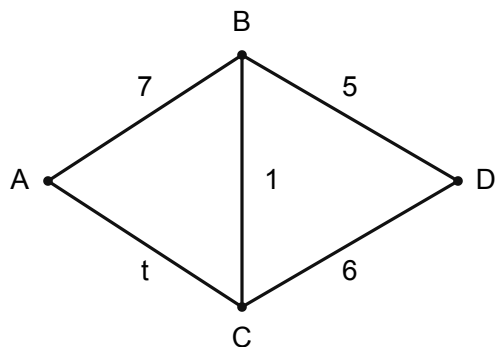
- 1** Travel times are minutes. Carry equipment from Media Room M to Gym G. Which route is fastest? Show all route totals.



- 2** Cables cost the edge weights in dollars. Connect all four rooms using a spanning tree of minimum cost. Give its three edges and justify.

**CHALLENGE** One step up

Weights are minutes and $AC = t > 0$. For what t is A-B-D a shortest route? Include ties and compare all four route formulas.



PRACTICE 1

A-B-D: 12; A-B-C-D: 14; A-C-B-D: 16; A-C-D: 16. Minimum 12.

PRACTICE 2

W-X-Z: 21; W-X-Y-Z: 22; W-Y-Z: 23; W-Y-X-Z: 32. Minimum 21.

PRACTICE 3

L-M-O: 11; L-M-N-O: 14; L-N-O: 16; L-N-M-O: 27. Minimum 11.

PRACTICE 4

No. A-C-D = 11; A-B-C-D = 12; A-B-D = 13; A-C-B-D = 20. Greedy edge choice alone fails.

APPLY IT 1

M-A-G, 17 minutes.

M-A-G = 17; M-C-G = 18; M-A-C-G = 18; M-C-A-G = 27.

APPLY IT 2

AB, BC, CD; cost \$6.

The three cheapest edges cost $1 + 2 + 3 = 6$ and connect all rooms without a cycle. Every spanning tree needs three edges, so no lower total is possible.

CHALLENGE

A-B-D = 12; A-C-D = $t + 6$; A-B-C-D = 14; A-C-B-D = $t + 6$. Thus $t \geq 6$; when $t = 6$ there are three tied shortest routes.

Any ordering of the four simple routes is acceptable. A path uses only existing edges. In the challenge, include equality at $t = 6$.