

Triangle Match Moves

Explain triangle congruence by moving, turning and reflecting.

HIGH SCHOOL · HSG-CO.B.8

CCSS.Math.Content.HSG-CO.B.8 "Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions."

NAME _____

DATE _____

Materials and setup

Pencil and ruler. Drawings illustrate reasoning; measurement is not proof. Assume nondegenerate triangles and the stated correspondence.

CONGRUENCE MEANS AN EXACT FIT

Two figures are congruent when a sequence of rigid motions maps one onto the other. Translations, rotations and reflections preserve distances and angle measures. For triangles ABC and DEF , use the correspondence A to D , B to E and C to F . A valid explanation shows why the third vertex must match after a corresponding side is placed; naming SSS, SAS or ASA alone is not enough.

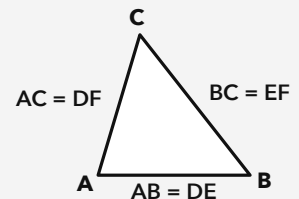
WORKED EXAMPLE

Explain SSS from rigid motions: $AB = DE$, $AC = DF$ and $BC = EF$. Why must the triangles match? The sketch labels the matching side data; the argument applies to any nondegenerate triangle with these equalities.

1. Translate A onto D , then rotate the image of AB onto ray DE . Equal lengths put B onto E .
2. The image of C must be distance AC from D and distance BC from E . For an existing nondegenerate triangle, two circles with those centers and radii meet at two points, one on each side of DE .
3. F is one of those points. The image of C is F or its reflection across DE . If needed, reflect across DE , fixing D and E and sending the third vertex to F .
4. Thus one sequence of rigid motions maps all three corresponding vertices and the whole triangle. This proves SSS; it does not assume SSS.

Answer:

SSS follows because the two distance constraints fix the third vertex up to reflection across the matched side.



PRACTICE 4 problems

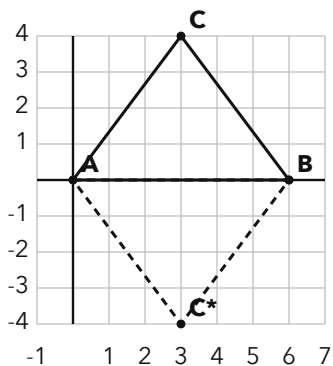
- 1** Translate $P(0,0)$, $Q(2,0)$, $R(0,1)$ by $(3,2)$. Give all image coordinates. Which features does the translation preserve?

ANSWER _____

- 2** In the SSS argument, why does rotating AB onto ray DE place B at E instead of somewhere else on that ray?

ANSWER _____

- 3** The grid shows $A(0,0)$, $B(6,0)$, $C(3,4)$ and $C^*(3,-4)$. Find AC , BC , AC^* and BC^* . Which rigid motion fixes A and B and exchanges C with C^* ?



ANSWER _____

- 4** A student doubles every side of a triangle and says the result is congruent because the angles match. Explain the error.

ANSWER _____

APPLY IT Show your work

Derive SAS. Given $AB = DE$, $AC = DF$ and $\angle BAC = \angle EDF$, first map A to D and B to E.

- 1** Explain why a reflection across DE may be needed, then why the image of C must equal F. Do not use SAS itself as a reason.

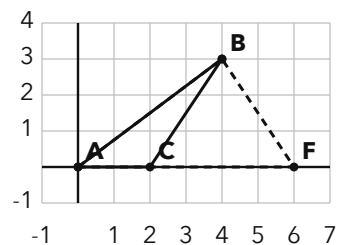
ANSWER _____

Derive ASA. Given $AB = DE$, $\angle BAC = \angle EDF$ and $\angle ABC = \angle DEF$, map A to D and B to E. Explain how a possible reflection and the two endpoint angles force the image of C to coincide with F. Do not cite ASA as a reason.

ANSWER _____

CHALLENGE One step up

Test SSA with $A(0,0)$, $B(4,3)$, $C(2,0)$ and $F(6,0)$. Compare triangles ABC and ABF . Show that AB is shared, $BC = BF$, and angle $BAC =$ angle BAF , but the triangles are not congruent. Why does this disprove SSA as a general criterion?



PRACTICE 1

Translate $P(0,0)$, $Q(2,0)$, $R(0,1)$ by $(3,2)$. Give all image coordinates. Which features does the translation preserve?

$P(3,2)$, $Q(5,2)$, $R(3,3)$. It preserves all distances and angle measures.

PRACTICE 2

In the SSS argument, why does rotating AB onto ray DE place B at E instead of somewhere else on that ray?

The rotation places B on ray DE , and $AB = DE$ fixes the distance from D . There is one point on that ray at that distance.

PRACTICE 3

The grid shows $A(0,0)$, $B(6,0)$, $C(3,4)$ and $C^*(3,-4)$. Find AC , BC , AC^* and BC^* . Which rigid motion fixes A and B and exchanges C with C^* ?

All four distances are 5. Reflection across the x -axis (line AB) fixes A and B and exchanges the third vertices.

PRACTICE 4

A student doubles every side of a triangle and says the result is congruent because the angles match. Explain the error.

Doubling lengths changes distances. It is a dilation, not a rigid motion; equal angles alone do not guarantee congruence.

APPLY IT 1

After the first motions, B is at E because $AB = DE$. Reflect across DE if necessary so the image of C and F lie in the same half-plane. The equal included angles place ray AC on ray DF . The equality $AC = DF$ fixes the third vertex at F on that ray. Therefore all vertices match by rigid motions.

The half-plane choice resolves the two possible sides of the base. On the chosen side, one ray makes the given angle with DE , and one point on that ray lies at the required distance from D .

APPLY IT 2

Reflect across DE if needed so the third vertices are on the same side. The angle at D fixes the ray from D toward the third vertex. The angle at E fixes the ray from E toward it. These two rays intersect at the unique point F , so the image of C is F . All corresponding vertices match by rigid motions.

The included side fixes the endpoints before the two angle constraints fix the rays. The existing nondegenerate triangle ensures that the rays meet at one point, not along a line.

CHALLENGE

$AB = 5$ for both triangles. BC and BF each have length square root of 13, since their squared lengths are $2^2 + 3^2$. Rays AC and AF are the same positive horizontal ray, so the angles at A match. But $AC = 2$ and $AF = 6$, and the side-length sets differ; the triangles cannot be congruent. These same SSA data allow two different triangles, so SSA is not a general guarantee. Some specific SSA data can still determine one triangle.

Require connected rigid-motion explanations of all three criteria, including the role of reflection and why the third vertex is fixed. The SSS grid illustrates two reflected positions; the general worked proof uses circle intersections rather than only one measured case. The SSA counterexample has side lengths $\{5, \sqrt{13}, 2\}$ and $\{5, \sqrt{13}, 6\}$; no vertex relabeling makes these sets equal. Accept equivalent valid reasoning. No target congruence criterion may be used to prove itself.