

Volume Power

Explain where volume formulas come from, then use them.

HIGH SCHOOL · HSG-GMD.A

CCSS.Math.Content.HSG-GMD.A "Explain volume formulas and use them to solve problems"

NAME _____

DATE _____

CROSS SECTIONS CONNECT THE FORMULAS

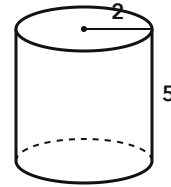
Stacking constant area B through perpendicular height h gives $V = Bh$.
Cavalieri's principle: equal heights and equal cross-sectional areas at every matching level imply equal volumes. Lengths are in cm; figures are schematic.
Keep π exact.

WORKED EXAMPLE

Explain the volume of a cylinder with radius 2 and height 5.

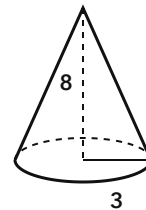
1. Slices parallel to the base have area $\pi(2^2) = 4\pi \text{ cm}^2$.
2. Stack $4\pi \text{ cm}^2$ through 5 cm: $V = Bh = 20\pi \text{ cm}^3$.

Answer: $20\pi \text{ cm}^3$. Area times height gives cubic units.



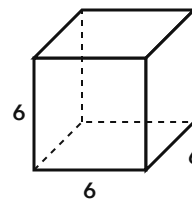
PRACTICE 4 problems

- 1 A cone has radius 3 and perpendicular height 8. A matching cylinder has the same base and height. A filling experiment takes three full cones to fill that cylinder. Use this comparison to explain the cone formula and find both volumes.



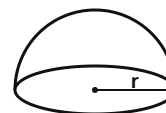
ANSWER _____

- 2** A cube of side 6 can be divided into three congruent square pyramids. They share one cube vertex as their apex; their bases are the three cube faces that do not contain that vertex. Explain the volume of one pyramid using the cube, then express it as $Bh/3$.



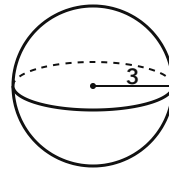
ANSWER

- 3** For a hemisphere of radius r , the slice at height z above its flat base has area $\pi(r^2 - z^2)$. Compare a radius- r cylinder of height r with a cone removed. The cone has vertex at $z = 0$ and base radius r at $z = r$, so its slice radius at height z is z . Use these slice areas, Cavalieri, and $V_{\text{cone}} = Bh/3$ to derive the full sphere's volume formula.



ANSWER

- 4** A sphere has radius 3. Find its volume. If its radius doubles, explain why the volume is multiplied by 8 rather than 2, using the formula you derived.



ANSWER _____

APPLY IT Show your work

- 1** A storage model consists of a cylinder of radius 4 and height 9, with a cone of the same radius and height 3 on top. The interiors meet at one circular face and do not overlap. Find the total volume; explain why each term has its chosen factor.

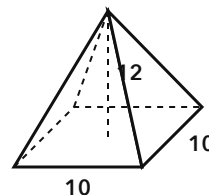
ANSWER _____

2

A solid square pyramid has base side 10 and perpendicular height 12.

A designer claims its volume is $1,200 \text{ cm}^3$ because $B = 100$ and $h = 12$.

Explain the missing factor and find the correct volume. Compare it with a prism of that base and height.



ANSWER _____

CHALLENGE

One step up

A right cylinder and a slanted cylinder both have horizontal circular bases of radius 3 and perpendicular height 7. At every horizontal level, both cross sections are circles of radius 3. Explain why their volumes match and calculate the volume. Why should the longer slanted side NOT replace 7 in Bh ?

PRACTICE 1

The cylinder holds $\pi(3^2)(8) = 72\pi \text{ cm}^3$. Three cone volumes equal Bh , so $V_{\text{cone}} = Bh/3 = 24\pi \text{ cm}^3$. The experiment illustrates the one-third relationship; it is not a general proof.

PRACTICE 2

The cube has volume $6^3 = 216 \text{ cm}^3$. Each congruent pyramid has one third: 72 cm^3 . Its base area is 36 cm^2 and perpendicular height is 6 cm , so $Bh/3 = 36(6)/3 = 72 \text{ cm}^3$.

PRACTICE 3

The hemisphere equals the cylinder minus the cone: $\pi r^3 - \pi r^3/3 = 2\pi r^3/3$. Doubling gives $V_{\text{sphere}} = 4\pi r^3/3$. The compared solids have height r and equal slice areas at each $0 \leq z \leq r$.

PRACTICE 4

$V = (4/3)\pi(3^3) = 36\pi \text{ cm}^3$. Doubling gives $(4/3)\pi(6^3) = 288\pi \text{ cm}^3$. The radius is cubed: $(2r)^3 = 8r^3$.

APPLY IT 1

$160\pi \text{ cm}^3$.

The cylindrical part is $Bh = \pi(4^2)(9) = 144\pi$. The conical part is $Bh/3 = \pi(4^2)(3)/3 = 16\pi$. Add the disjoint interior volumes: $160\pi \text{ cm}^3$.

APPLY IT 2

400 cm^3 , one third of the matching prism's $1,200 \text{ cm}^3$.

A pyramid tapers; its slices are not all area B . Its volume is $Bh/3 = 100(12)/3 = 400 \text{ cm}^3$. The formula Bh alone applies to the matching prism.

CHALLENGE

Cavalieri gives equal volumes because corresponding slice areas and perpendicular heights match. $V = \pi(3^2)(7) = 63\pi \text{ cm}^3$ for each. The stack thickness is measured perpendicular to the bases; the slanted side does not measure that thickness.

Require reasons as well as calculations. In Practice 2, the three cube pyramids have bases on the three faces that do not contain the common opposite vertex. In Practice 3, the cone's vertex is at $z = 0$ and its radius at level z is z , so its slice area is πz^2 . The supplied equal-slice relation makes this an informal Cavalieri argument.