

Residual Reality Check

Calculate, plot, and interpret residuals to assess fit.

HIGH SCHOOL • HSS-ID.B.6b

CCSS.Math.Content.HSS-ID.B.6b "Informally assess the fit of a function by plotting and analyzing residuals."

NAME _____

DATE _____

PLOT THE ERRORS, NOT THE ORIGINAL Y-VALUES

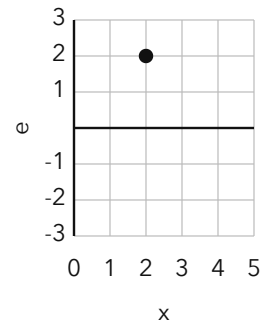
Residual $e = \text{actual } y - \text{predicted } y$. Plot each (x, e) as a separate dot; the horizontal axis is the zero-residual line. Positive e means underprediction, negative e means overprediction. Check patterns AND error size in the original units. A small sample can suggest a useful model, not prove it will always work.

WORKED EXAMPLE

A model predicts $y = 3x + 5$ centimeters. At $x = 2$ the actual length is 13 cm. Calculate and plot its residual.

1. Predicted $y = 3(2) + 5 = 11$ cm. The residual is $13 - 11 = 2$ cm.
2. Plot $(2, 2)$, not $(2, 13)$. The point lies above zero; the model underpredicts by 2 cm.

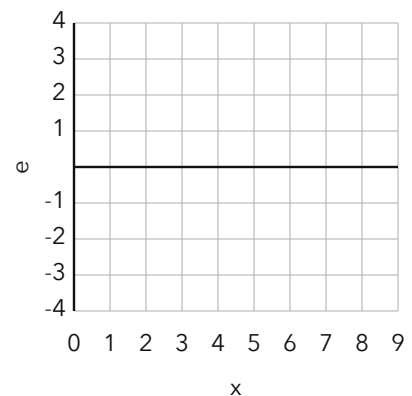
Answer: $e = +2$ cm. One observation alone cannot establish overall fit.



PRACTICE 4 problems

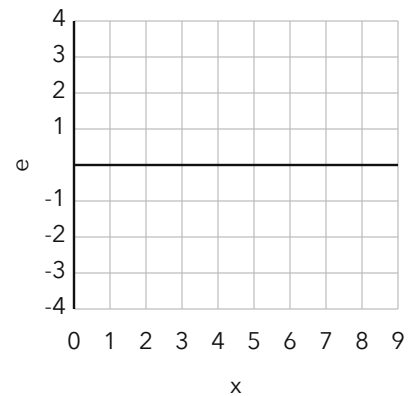
- 1** For $x = 1$ through 8 in order, measured lengths y in cm are 13, 12, 16, 19, 19, 22, 26, 25. A model predicts $y = 2x + 10$. Calculate all residuals, plot them, and assess the pattern and maximum error.

ANSWER _____



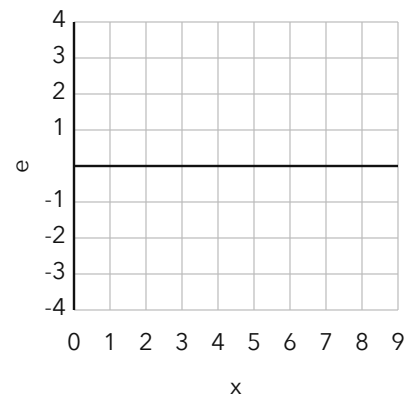
- 2** A second length model gives residuals $(x, e \text{ in cm})$: $(1, -4), (2, -3), (3, -2), (4, -1), (5, 0), (6, 1), (7, 2), (8, 3)$. Plot them. Explain what the sign change and pattern suggest about the model.

ANSWER



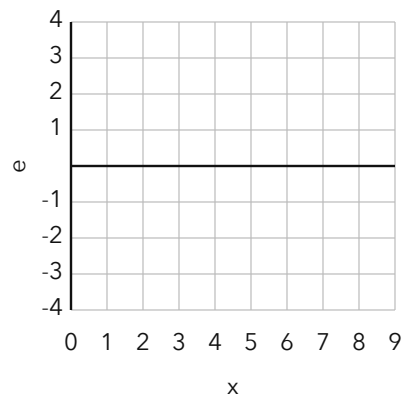
- 3** A linear model gives $(1, 3), (2, 1), (3, -1), (4, -2), (5, -2), (6, -1), (7, 1), (8, 3)$, with e in cm. Plot the residuals. Describe the shape and explain whether a curved model is worth investigating.

ANSWER



- 4** A machine-time model gives $(1, 3), (2, -4), (3, 1), (4, 4), (5, -2), (6, 2), (7, -3), (8, 0)$, with e in MINUTES. Plot them. No clear curve is apparent. If errors over 2 minutes are unacceptable, is this model adequate for scheduling? Explain.

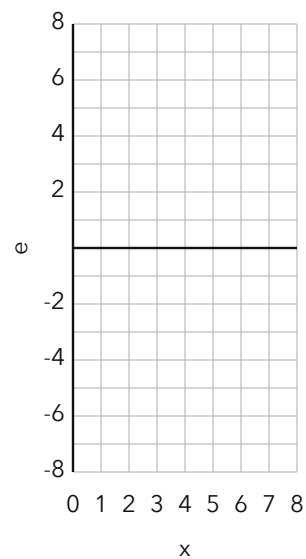
ANSWER



APPLY IT Show your work

- 1** A test device is observed at settings $x = 1, 2, 3, 4, 5, 6, 7$. Its output y in units is 41, 41, 40, 44, 50, 55, 65, respectively. Model A is $y = 4x + 30$. Calculate all seven residuals and plot them. Assess the pattern.

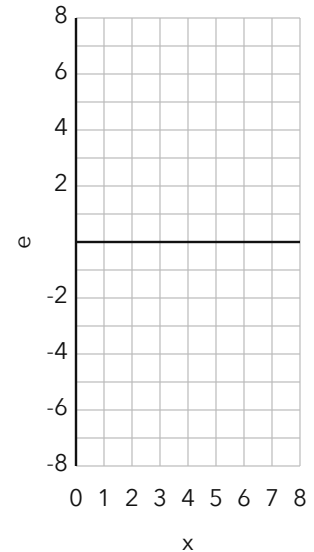
ANSWER



2

For the SAME seven device observations above, Model B is $y = x^2 - 4x + 44$. Calculate and plot its residuals. Compare its fit with Model A over these measured settings; use both pattern and error size.

ANSWER _____



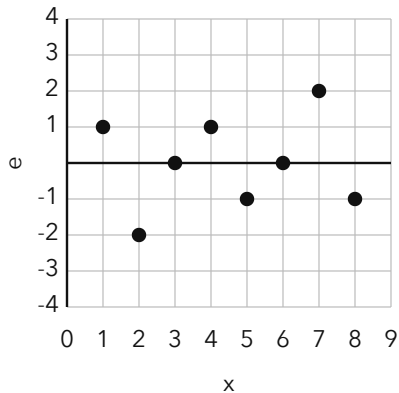
CHALLENGE

One step up

A student says, 'Model B's residuals are small, so it must work well at $x = 50$.' Use the plotted evidence to respond. Name one check you would make before using B outside the measured range.

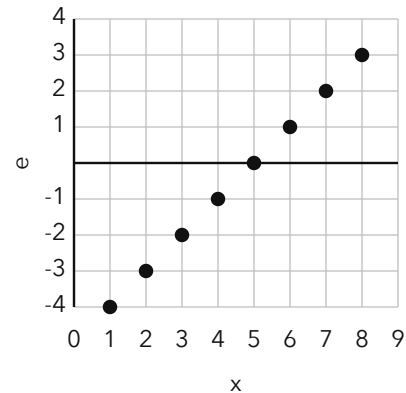
PRACTICE 1

Residuals: 1, -2, 0, 1, -1, 0, 2, -1 cm. They have no clear trend or curve and maximum absolute error 2 cm. This suggests a reasonable fit over these observations if that error is acceptable for the purpose.



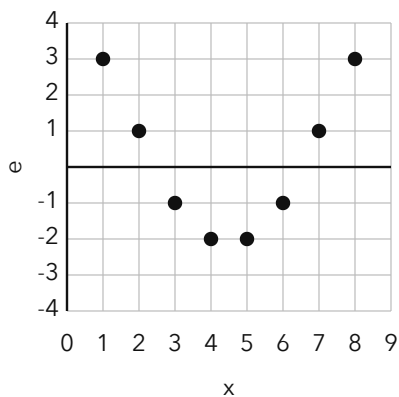
PRACTICE 2

Residuals rise systematically from -4 to 3 cm. The model overpredicts at smaller x and underpredicts at larger x, suggesting its slope is too small for this range. The trend argues against accepting the fit unchanged.



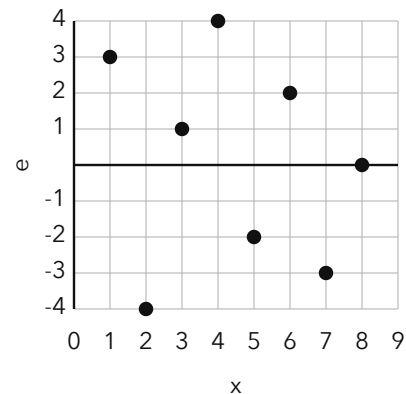
PRACTICE 3

A U-shaped pattern: positive residuals at the ends and negative residuals in the middle. The line misses systematic curvature; investigate a curved model and validate its fit rather than concluding that any curve is automatically better.



PRACTICE 4

No. Several errors exceed 2 minutes in magnitude, up to 4 minutes. Lack of an obvious curve is insufficient when the errors are too large for the scheduling purpose.

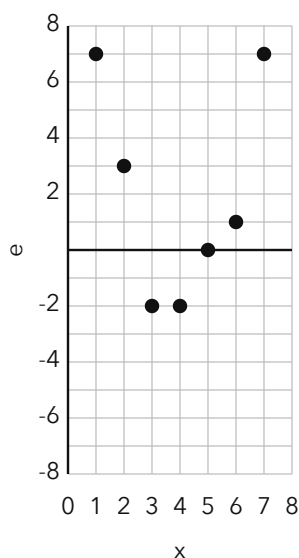


APPLY IT 1

Residuals: 7, 3, -2, -2, 0, 1, 7 units.

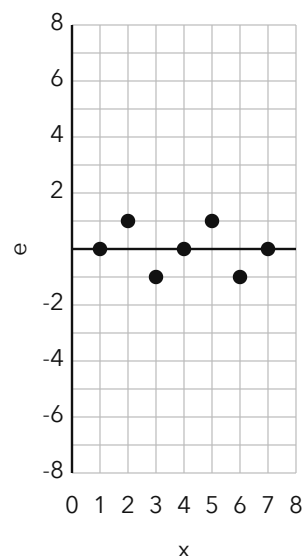
Subtract predictions 34, 38, 42, 46, 50, 54, 58.

Residuals tend to be positive at the ends and lower in the middle; the curved pattern suggests this straight-line model misses structure.

**APPLY IT 2**

Residuals: 0, 1, -1, 0, 1, -1, 0 units. B fits these observations better.

Predictions are 41, 40, 41, 44, 49, 56, 65. B's errors are at most 1 unit, versus A's 7, with no broad curved pattern remaining. A small repeating sequence alone is weak evidence with seven points; gather further data before claiming general adequacy.

**CHALLENGE**

The evidence covers settings 1-7 only. Small residuals there do not establish accuracy at 50. Collect observations at new settings, compare predictions with measurements, and examine residual pattern and size in those units. Also check whether the device can operate in that range.

Use actual minus predicted consistently; do not connect residual dots. Judge broad systematic patterns and practical error tolerance. Finite samples do not prove random errors or reliable extrapolation. The two device plots use the same axes so visual comparison is fair. A residual at zero is still an observation and must be plotted.