

# Zero In

Use polynomial theorems to find remainders, factors, and zeros.

HIGH SCHOOL • MA.912.AR.6.3

**MA.912.AR.6.3** "Explain and apply theorems for polynomials to solve mathematical and real-world problems."

NAME \_\_\_\_\_

DATE \_\_\_\_\_

## FACTORS AND ALL ZEROS

Factor Theorem:  $x-r$  is a factor of  $P(x)$  exactly when  $P(r)=0$ . The Fundamental Theorem of Algebra implies a degree- $n$  polynomial has  $n$  complex zeros, counting multiplicity. This does not mean  $n$  distinct or real zeros. For real coefficients, nonreal zeros occur in conjugate pairs. Here  $i^2=-1$ ; monic means leading coefficient 1.

## WORKED EXAMPLE

Solve  $P(x)=x^3-2x^2+x-2=0$  over the complex numbers. Explain why the zero list is complete.

1.  $P(2)=8-8+2-2=0$ ; the Factor Theorem gives factor  $x-2$ .
2.  $P(x)=(x-2)(x^2+1)$ . Solve  $x^2=-1$  to get  $i$  and  $-i$ .
3. There are three zeros counting multiplicity, matching degree 3.

**Answer: Zeros 2,  $i$ ,  $-i$ , each simple. The degree-3 count proves completeness.**

## PRACTICE 4 problems

- 1** Use the Factor Theorem to test  $x=3$  for  $P(x)=x^3-3x^2+4x-12$ . Then factor  $P$  and find all complex zeros.

**ANSWER** \_\_\_\_\_  
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- 2** For  $P(x)=(x-1)^2(x+2)(x^2+9)$ , list each distinct zero and its multiplicity. State degree, total zero count and number of distinct zeros.

**ANSWER** \_\_\_\_\_  
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- 3** A monic cubic with real coefficients has zeros  $-1$  and  $2+i$ . Find its third zero and write the polynomial in factored and expanded forms.

**ANSWER** \_\_\_\_\_  
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- 4** A real-coefficient quartic has zeros  $0, 2, 1+i$ . Identify the remaining zero. Could it have a fifth distinct zero? Explain.

**ANSWER** \_\_\_\_\_  
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## APPLY IT Show your work

- 1** A box has dimensions  $x$ ,  $x-1$  and  $x-2$  centimeters and volume  $24 \text{ cm}^3$ . Test  $x=4$  in  $x^3-3x^2+2x-24=0$ , then factor it. Explain why one usable length does not contradict the degree-3 zero count.

WORK SPACE

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**ANSWER** \_\_\_\_\_

- 2** A design condition is  $x^3-6x^2+13x-10=0$ , where  $x$  is a positive length in meters. Test  $x=2$ , find all complex solutions, and identify the usable length.

WORK SPACE

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**ANSWER** \_\_\_\_\_

## **CHALLENGE** One step up

A real-coefficient polynomial has zeros 3 with multiplicity 2 and  $1-2i$ . Find its smallest possible degree and write the monic polynomial of that degree. Explain why the zero list is complete.

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WORK SPACE AND ANSWER

**PRACTICE 1**

Use the Factor Theorem to test  $x=3$  for  $P(x)=x^3-3x^2+4x-12$ . Then factor  $P$  and find all complex zeros.

**$P(3)=0$ ;  $P=(x-3)(x^2+4)$ ; zeros  $3, 2i, -2i$ , all simple.**

**PRACTICE 2**

For  $P(x)=(x-1)^2(x+2)(x^2+9)$ , list each distinct zero and its multiplicity. State degree, total zero count and number of distinct zeros.

**Degree 5; zeros 1 (multiplicity 2),  $-2, 3i, -3i$  (each 1). Total 5 counting multiplicity; 4 distinct.**

**PRACTICE 3**

A monic cubic with real coefficients has zeros  $-1$  and  $2+i$ . Find its third zero and write the polynomial in factored and expanded forms.

**Third zero  $2-i$ ;  $P=(x+1)(x^2-4x+5)=x^3-3x^2+x+5$ .**

**PRACTICE 4**

A real-coefficient quartic has zeros  $0, 2, 1+i$ . Identify the remaining zero. Could it have a fifth distinct zero? Explain.

**Remaining zero  $1-i$ . No fifth distinct zero: a degree-4 polynomial has exactly 4 complex zeros counting multiplicity.**

**APPLY IT 1**

**$P(4)=0$ ;  $P=(x-4)(x^2+x+6)$ . The quadratic has discriminant  $-23$  and two nonreal zeros. Only  $x=4$  is physical, giving 4 cm by 3 cm by 2 cm.**

Require  $x>2$ . There are three complex zeros counting multiplicity, but just one real dimension.

**APPLY IT 2**

**$P(2)=0$ ;  $P=(x-2)(x^2-4x+5)$ . Zeros  $2, 2+i, 2-i$ ; usable length 2 m.** The quadratic equals  $(x-2)^2+1$ . Nonreal roots cannot represent a physical length.

**CHALLENGE**

**Degree 4; conjugate zero  $1+2i$ .**

**$P=(x-3)^2((x-1)^2+4)=x^4-8x^3+26x^2-48x+45$ .**

**Multiplicities  $2+1+1=4$  match the degree.**

Require explicit theorem-based explanations. Count multiplicity, distinguish distinct from total zeros, and include nonreal roots when solving over the complex numbers. Conjugate pairing requires real coefficients. Physical models retain only viable real solutions. Accept equivalent factorizations.