

Polynomial Graph Detective

Sketch polynomial graphs and spot their key features.

HIGH SCHOOL • MA.912.AR.6.4

MA.912.AR.6.4 "Given a table, equation or written description of a polynomial function of degree 3 or higher, graph that function and determine its key features."

NAME _____

DATE _____

READ THE FACTORS

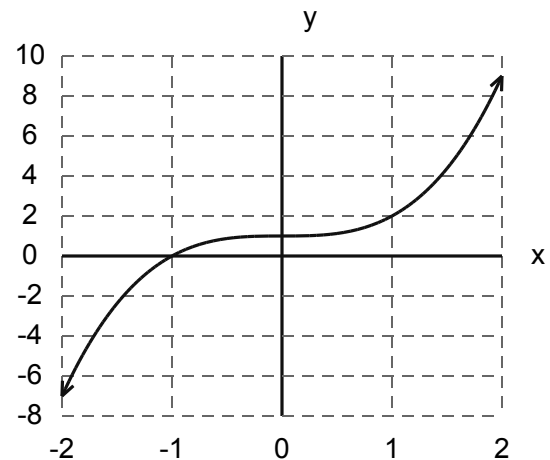
Use zeros, multiplicities and additional points to sketch a smooth polynomial graph. Even-multiplicity zeros touch; odd-multiplicity zeros cross. Identify domain/range, intercepts, intervals of increase/decrease and sign, relative extrema, symmetry and end behavior. R means all real numbers. A plotting window does not restrict an unrestricted function.

WORKED EXAMPLE

Graph $y=x^3+1$. Give intercepts, domain/range, monotonic behavior and end behavior.

1. Plot $(-1,0),(0,1),(1,2)$; add points to reveal the cubic shape.
2. The function increases over all real x ; the flat point at $(0,1)$ is not a maximum or minimum.
3. Continue down to the left and up to the right.

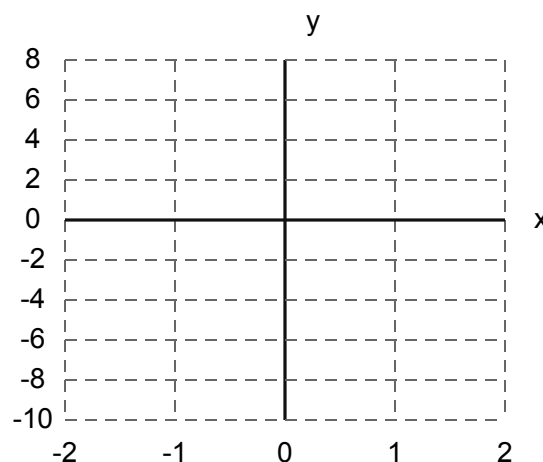
Answer: Intercepts $(-1,0),(0,1)$; domain and range $\mathbb{R}$; increasing throughout; left end down, right end up.



PRACTICE 4 problems

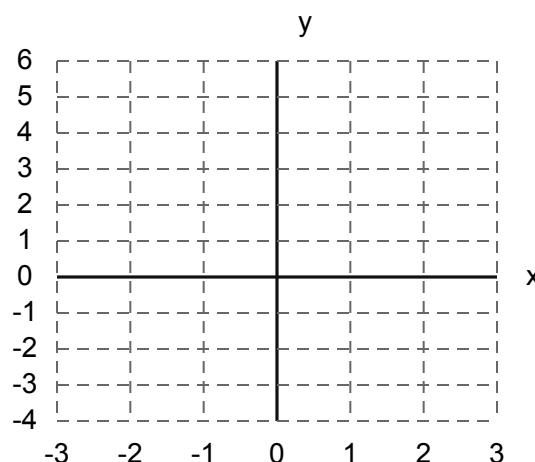
- 1** Graph $y=x^3-1$. Give intercepts, domain/range and end behavior. Does its flat point at $x=0$ make it a relative extremum?

ANSWER _____



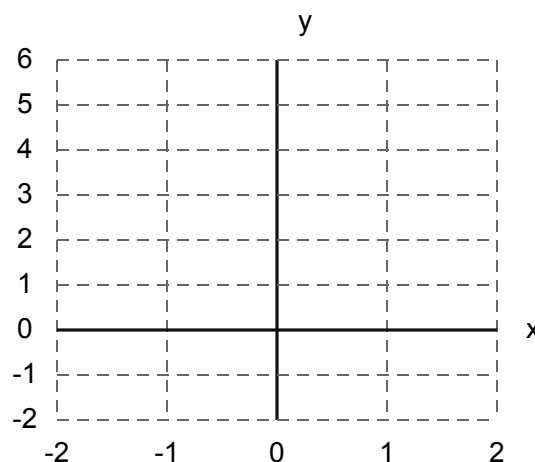
- 2** Graph $y=-(x+1)^2(x-2)$. Identify intercepts, relative extrema and intervals of increase/decrease.

ANSWER _____



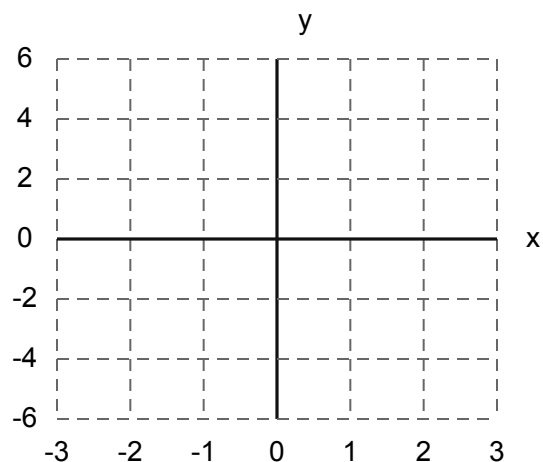
- 3** Graph $y=x^4-1$. Give domain/range, intercepts, symmetry, extrema and end behavior.

ANSWER _____



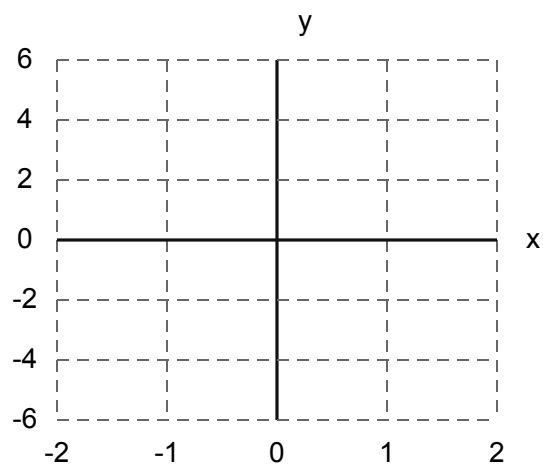
- 4** A monic cubic has zeros $-2, 0, 2$. Write and graph it.
State its y -intercept, symmetry and intervals where it is positive or negative.

ANSWER _____



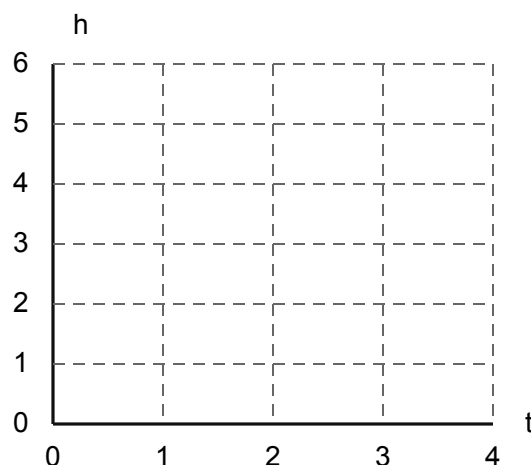
APPLY IT Show your work

- 1** A cubic has table $x = -2, -1, 0, 1, 2$ and $y = -6, 0, 0, 0, 6$. Find its equation and graph it. Give intercepts and end behavior.



2

A simulated object has height $h = -t^3 + 3t^2$ meters at t seconds, for $0 \leq t \leq 3$. Graph its motion. Find its maximum, ground times, domain/range and rising/falling intervals.

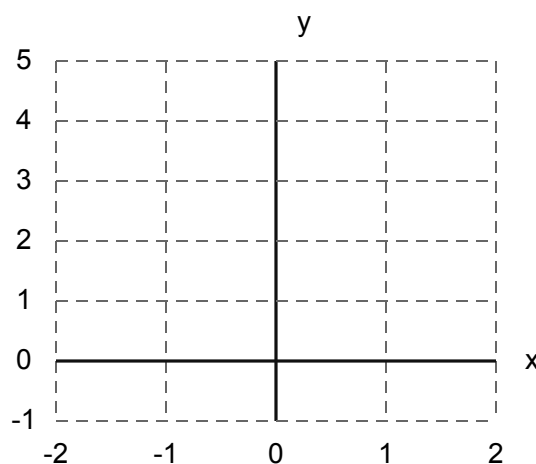


CHALLENGE

One step up

Graph $y = (x^2 - 1)^2$. Identify domain/range, zeros with touch/cross behavior, all relative extrema, increasing/decreasing intervals, symmetry and end behavior.

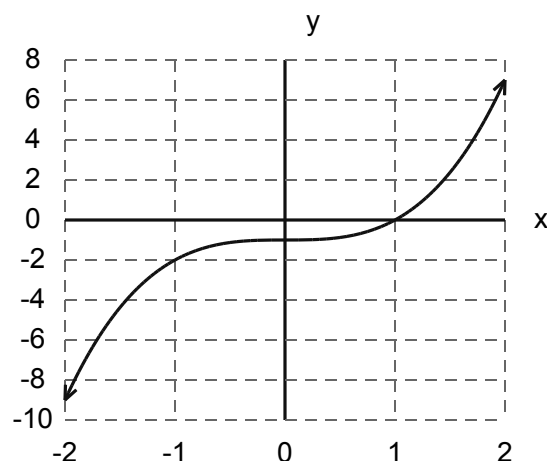
WORK SPACE AND ANSWER



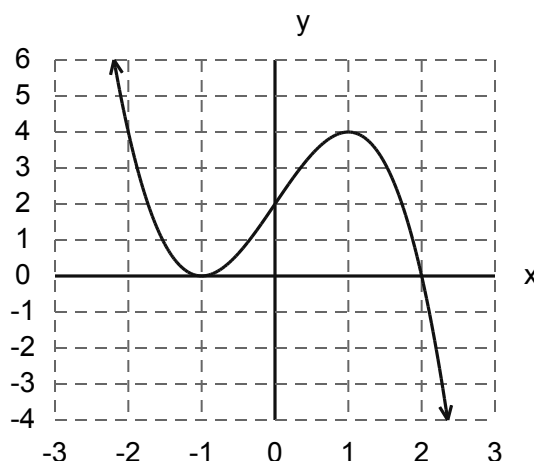
Require smooth graphs and the requested features, not only zeros and end arrows. A horizontal tangent need not be an extremum. Unrestricted polynomials have domain $\mathbb{R}$; the height model stops at its closed physical domain. Monic means leading coefficient 1. Accept sketches with accurately placed intercepts and extrema.

PRACTICE 1

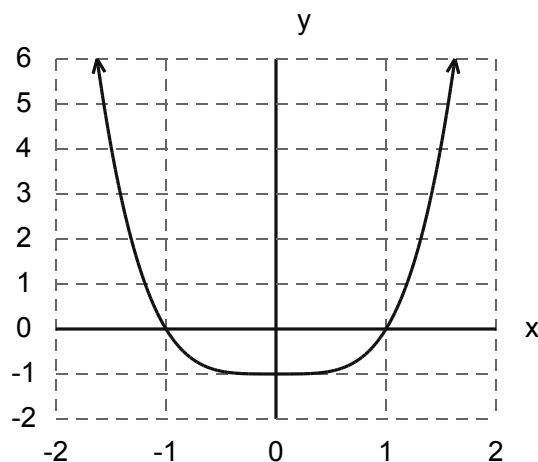
Intercepts $(1,0), (0,-1)$; domain/range $\mathbb{R}$; left down, right up. No extremum at $x=0$: the function increases through it.

**PRACTICE 2**

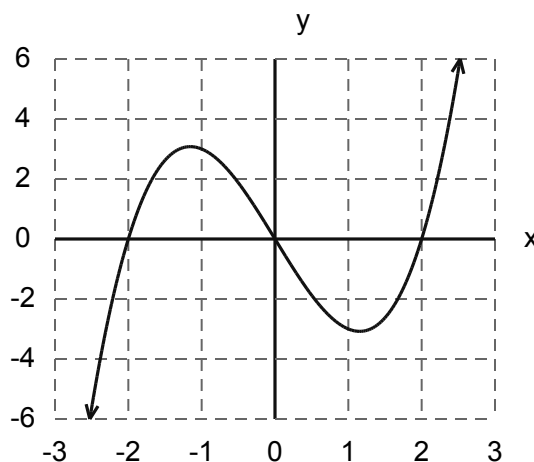
Intercepts $(-1,0), (2,0), (0,2)$. Relative min $(-1,0)$, max $(1,4)$. Increasing $-1 < x < 1$; decreasing $x < -1$ or $x > 1$.

**PRACTICE 3**

$D=\mathbb{R}$; range $y \geq -1$; intercepts $(-1,0), (1,0), (0,-1)$. Symmetric about y-axis; minimum $(0,-1)$; both ends rise.

**PRACTICE 4**

$y=x(x+2)(x-2)=x^3-4x$; y-intercept $(0,0)$; origin symmetry. Positive $-2 < x < 0$ or $x > 2$; negative $x < -2$ or $0 < x < 2$.

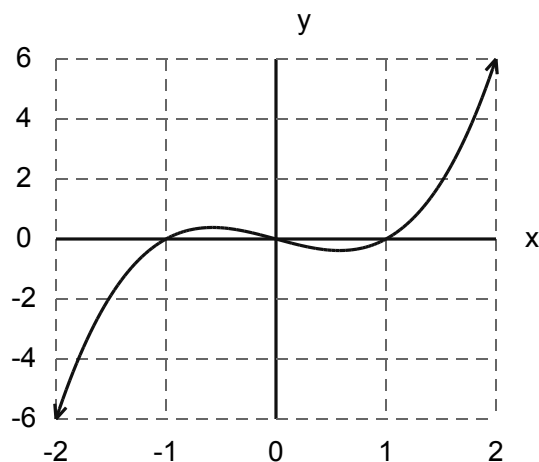


APPLY IT 1

$y = x^3 - x$; intercepts $(-1,0), (0,0), (1,0)$, all crossing.

Left end down, right end up.

The three zeros give $a \cdot x \cdot (x+1) \cdot (x-1)$; use $(2,6)$ to find $a=1$.

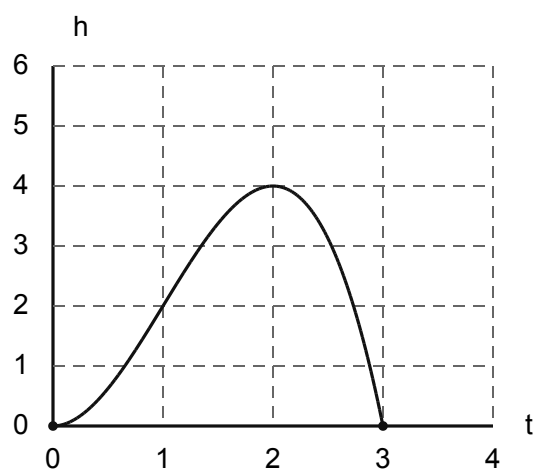


APPLY IT 2

Maximum 4 m at $t=2$ s; ground at $t=0,3$ s.

$D: 0 \leq t \leq 3$; range $0 \leq h \leq 4$. Rising $0 < t < 2$; falling $2 < t < 3$.

$h = t^2(3-t)$; both ground times are included, and the model ends at $t=3$.



CHALLENGE

$D=\mathbb{R}$; $y \geq 0$; zeros $-1, 1$ both touch. Minima $(-1,0), (1,0)$, max $(0,1)$. Increasing $-1 < x < 0$ or $x > 1$; decreasing $x < -1$ or $0 < x < 1$. Y-axis symmetry; both ends rise.

