Angle Evidence Lab

Build arguments that explain why angle relationships hold.

GRADE 8 · TEKS 8.8.D

8.8.D "use informal arguments to establish facts about the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle- angle criterion for similarity of triangles."

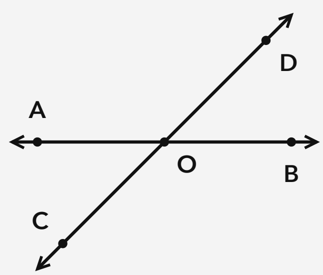
NAME ______________________ DATE ____________

STARTING FACTS

A straight angle is 180°. Rigid motions preserve angles. A dilation preserves angles and multiplies every length by one scale factor. Through a point outside a given line, exactly one line is parallel to that line. Use these starting facts and earlier results to build the arguments below; measuring one drawing is not a general proof. Write each argument in its blank box; use the answer line for your conclusion.

WORKED EXAMPLE

Lines AB and CD cross at O. Explain why opposite angles AOC and BOD are equal without measuring.

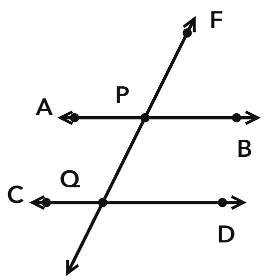


1. Angles AOC and COB form a straight angle, so their sum is 180°.
2. Angles COB and BOD also form a straight angle, so their sum is 180°.
3. Subtract the shared angle COB from both sums.

Answer: ∠AOC = ∠BOD; both equal 180° minus ∠COB.

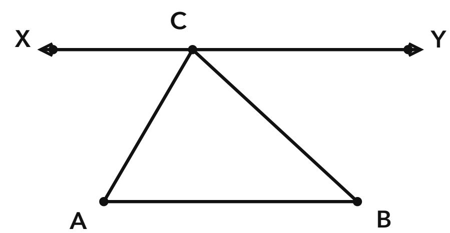
PRACTICE 4 problems

1 AB is parallel to CD; F, P and Q lie on the transversal. Slide the whole top intersection by the translation P to Q. Why must line AB land on CD? Use this to show ∠BPF = ∠DQP. Then use opposite angles to explain why the alternate interior angles APQ and DQP are equal.



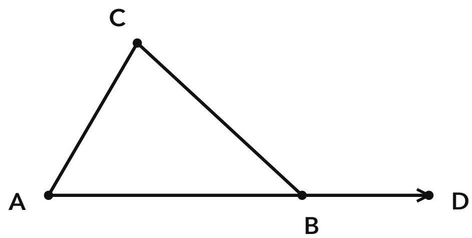
ANSWER ____________________

2 XY passes through C and is parallel to AB. Use the result of problem 1 to match ∠XCA with the angle at A, and ∠BCY with the angle at B. Explain why the three interior angles of triangle ABC sum to 180°. Do not assume the triangle-sum rule.



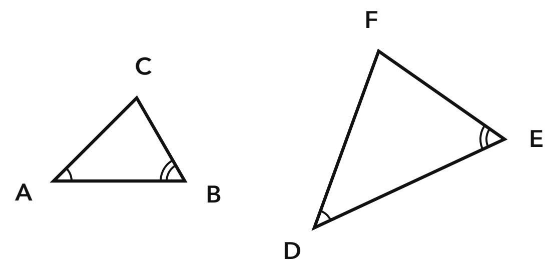
ANSWER ____________________

3 AB extends to D. Let a and c be the interior angles at A and C, b the interior angle at B, and e the exterior angle CBD. Write two equations using problem 2 and a straight angle. Subtract them to establish e = a + c.



ANSWER ______________________________

4 The marked angle pairs are equal: A matches D and B matches E. Move, turn and, if needed, flip ABC to align A with D and ray AB with ray DE. Dilate until B lands on E. Why must the rays to the third vertices coincide and meet at the same point? Explain why this establishes AA similarity, not necessarily congruence.



ANSWER ______________________________

APPLY IT Show your work

1 A triangular sign has two interior angles of 48° and 67°. Find the exterior angle at the third vertex. Justify using the third interior angle and a straight angle.

WORK SPACE

ANSWER ______________________

2 Roof triangles have angle pairs 50°, 60° and 50°, 70°. Find the missing angles, match vertices by angle, and explain why rigid motions and a dilation can align the triangles.

WORK SPACE

ANSWER ______________________

CHALLENGE One step up

Why can't measuring one triangle prove the 180° rule? Identify the general step in problem 2.

WORK SPACE AND ANSWER

PRACTICE 1

Translation sends AB to a line through Q parallel to AB, so it must be CD by uniqueness of the parallel. The transversal lands on itself. Thus corresponding angles BPF and DQP match. At P, APQ and BPF are opposite angles, so APQ also equals DQP.

PRACTICE 2

Alternate interior angles give ∠XCA = ∠CAB and ∠BCY = ∠ABC. Angles XCA, ACB and BCY together form straight angle XCY. Substituting the matching angles gives ∠CAB + ∠ACB + ∠ABC = 180°.

PRACTICE 3

a + b + c = 180° and b + e = 180°. Therefore a + b + c = b + e. Subtract b to obtain e = a + c: the exterior angle equals the two remote interior angles.

PRACTICE 4

The dilation matches AB to DE and preserves the angles at both endpoints. Equal angles at D put the third vertex on the same ray from D; equal angles at E put it on the same ray from E, on the chosen side of DE. Those two rays have one intersection, F. The transformed triangle matches DEF, so a rigid motion and dilation map ABC to DEF. This is similarity; the scale factor need not be 1.

APPLY IT 1

115° The third interior angle is 180 − 48 − 67 = 65°. The exterior angle forms a straight angle with it, so it is 180 − 65 = 115°. Subtracting the same third angle from 180 explains why the exterior equals 48 + 67.

APPLY IT 2

The missing angles are 70° and 60°; match 50° to 50°, 60° to 60°, and 70° to 70°. Each triangle has angles 50°, 60° and 70°. Match the side between the 50° and 60° vertices by a rigid motion and dilation. The equal endpoint angles then force the third vertex to the same intersection, as in problem 4. Side lengths may differ, so congruence is not established.

CHALLENGE

One measurement is an example and can have measurement error; it does not cover all triangles. For any nondegenerate triangle, a line through one vertex parallel to the opposite side produces two matching alternate interior angles. Together with the vertex angle they make a straight angle, so the reasoning is general.

Require connected reasons, not only a theorem name or a numerical result. Starting facts are printed; problem 1 establishes the parallel-angle relation used in problem 2, which establishes the triangle sum used in problem 3. For AA, students must explain the matched base and unique intersection after a dilation. Diagrams are schematic; compare marked angles, not apparent printed sizes.