

Function Shift Detectives

Find the change from graphs and tables.

HIGH SCHOOL • MA.912.F.2.3

MA.912.F.2.3 "Given the graph or table of $f(x)$ and the graph or table of $f(x)+k$, $kf(x)$, $f(kx)$ and $f(x+k)$, state the type of transformation and find the value of the real number k ."

NAME _____

DATE _____

MATCH INPUTS AND OUTPUTS

Use the specified form: $f(x)+k$ shifts outputs by k ; $kf(x)$ scales outputs, reflecting in the x -axis if $k<0$. $f(kx)$ divides each original x -coordinate by k and reflects in the y -axis if $k<0$. $f(x+k)$ shifts left by k . For $0<|k|<1$, vertical scaling compresses but horizontal scaling stretches. Track corresponding points; equal outputs alone need not identify equal inputs.

WORKED EXAMPLE

For $f(x)=x+1$, the table gives $g(x)=f(x)+k$. Find k and describe the shift.

MATCHED INPUTS

x	$(f(x), g(x))$
-1	(0, 3)
0	(1, 4)
1	(2, 5)

- At each common input, $g(x)-f(x)=3$.
- In the specified form $f(x)+k$, $k=3$: every point moves up 3.

Answer: $k=3$; vertical shift up 3 units.

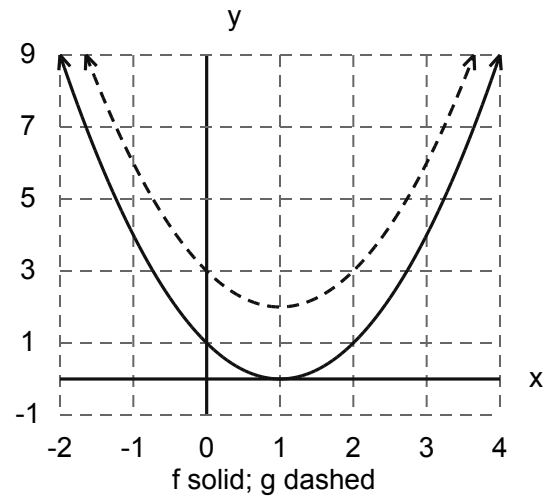
PRACTICE 4 problems

- 1** The table comes from $g(x)=k f(x)$. Find k , identify the transformation and explain why the row with $f(x)=0$ cannot determine k by division.

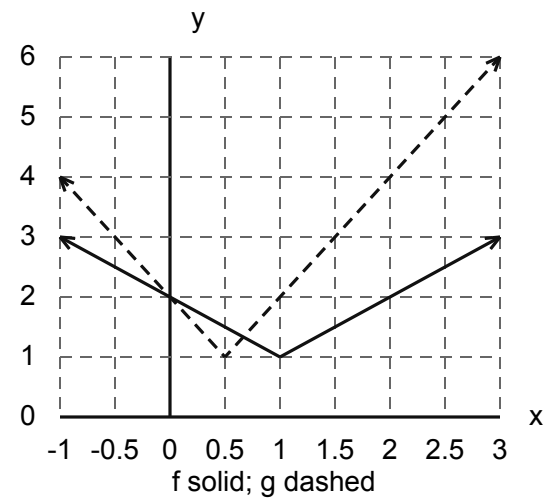
SAME INPUTS

x	$(f(x), g(x))$
-2	(-4, -2)
0	(0, 0)
2	(4, 2)

- 2 These are $f(x)=(x-1)^2$ and $g(x)=f(x)+k$. Find k from the graphs; verify with the vertex and one other point. Describe the transformation.



- 3 $f(x)=|x-1|+1$ and $g(x)=f(kx)$, $k>0$. Use the vertex x-coordinates to find k . Verify with the corresponding point (3,3) on f . State the horizontal scale factor.



- 4 Here $f(x)=x+1$ and $g(x)=f(x+k)$. Use the table to find k and the horizontal shift. Why is specifying $f(x+k)$ important for this line?

OBSERVED G

x	g(x)
0	-1
1	0
2	1
3	2

APPLY IT Show your work

1

A sensor reports $f(x)=2x+1$. A converter uses $g(x)=k f(x)$ and records these outputs. Find k and describe both the scale change and reflection of the graph. Verify using two rows.

CONVERTER MEASUREMENTS

x	(f(x), g(x))
-1	(-1, 2)
0	(1, -2)
1	(3, -6)

2

A display stretches the input axis of $f(x)=x+2$ using $g(x)=f(kx)$, $k>0$. The table lists known corresponding points, not merely pairs with equal heights. Find k and the horizontal scale factor.

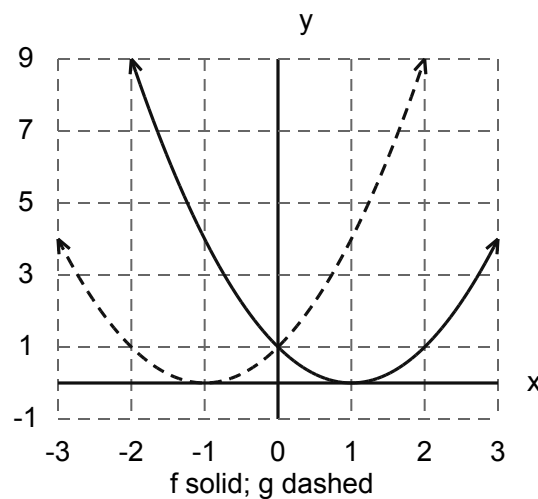
KNOWN POINT CORRESPONDENCE

Point on f	Point on g
(1,3)	(3,3)
(2,4)	(6,4)
(3,5)	(9,5)

CHALLENGE One step up

$f(x)=(x-1)^2$ and $g(x)=f(kx)$, k nonzero. Infer the sign and value of k from the vertices, then check $(0,1)$. If f were x^2 instead, could these kinds of graphs distinguish $k=1$ from $k=-1$?

WORK SPACE AND ANSWER



PRACTICE 1

$k=1/2$, a vertical compression by $1/2$. The nonzero rows give $g/f=1/2$; at zero, $0=k(0)$ holds for every k .

PRACTICE 2

$k=2$; up 2. Vertex $(1,0)$ becomes $(1,2)$; $(0,1)$ becomes $(0,3)$.

PRACTICE 3

The vertex $x=1$ becomes $x=1/2$, so $k=2$. $(3,3)$ maps to $(1.5,3)$. Horizontal compression by factor $1/2$.

PRACTICE 4

$k=-2$, a shift right 2. For example $g(2)=1=f(0)$, so $2+k=0$. The same line is also $f(x)-2$; the specified form tells us which parameter is being requested.

APPLY IT 1

$k=-2$: vertical stretch by 2 and reflection in the x -axis. At $x=0$, $-2/1=-2$; at $x=1$, $-6/3=-2$ (also $2/(-1)=-2$). Compare outputs at the same input; the multiplier is negative.

APPLY IT 2

Old input/new input $=1/3$ in each row, so $k=1/3$. Horizontal stretch by 3; $(1,3)$ becomes $(3,3)$. For $f(kx)$, k times the new input equals the original input.

CHALLENGE

Vertex input 1 becomes -1 , so $k=-1$: reflection in the y -axis. $(0,1)$ stays fixed. For $f=x^2$, $f(x)=f(-x)$, so the graphs could not distinguish those two signs.

Use only the transformation family named in each question. Horizontal scale factor is $1/|k|$, not $|k|$. A symmetric function may hide the sign of k ; a single shared output is not automatically a uniquely corresponding input.