

Function Shift Detectives

Decode transformations that happen together.

HIGH SCHOOL • MA.912.F.2.4

MA.912.F.2.4 "Given the graph or table of values of two or more transformations of a function, state the type of transformation and find the values of the real number that defines the transformation."

NAME _____

DATE _____

USE A FIXED TRANSFORMATION FORM

Unless another form is given, use $g(x)=a f(x-h)+k$, with no additional horizontal scale. A point (u,v) on f maps to $(u+h,av+k)$. Read the horizontal shift h first; then use two different original outputs to find a and k . Negative a reflects in the x -axis. Reflection and translation order matter. Tables marked correspondence specify which original and transformed points match.

WORKED EXAMPLE

For $f(x)=x^2$, these are known corresponding points under $g(x)=a f(x-h)+k$. Find all three parameters.

POINT CORRESPONDENCE

On f	On g
(0,0)	(1,3)
(1,1)	(2,5)
(2,4)	(3,11)

- Each input increases by 1, so $h=1$. The zero output becomes 3, so $k=3$.
- Use (1,1) to (2,5): $5=a+3$, so $a=2$. Check $11=2(4)+3$.

Answer: $g(x)=2f(x-1)+3$: right 1, vertical stretch by 2, up 3.

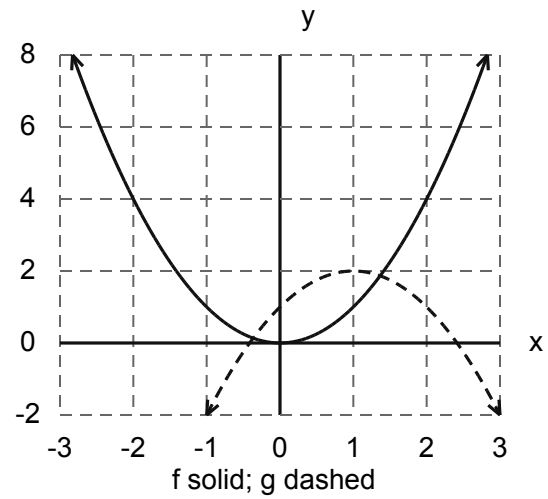
PRACTICE 4 problems

- 1** The table lists known corresponding points of $f(x)=|x|$ and $g(x)=a f(x-h)+k$. Find the parameters and describe all changes. Check the last row.

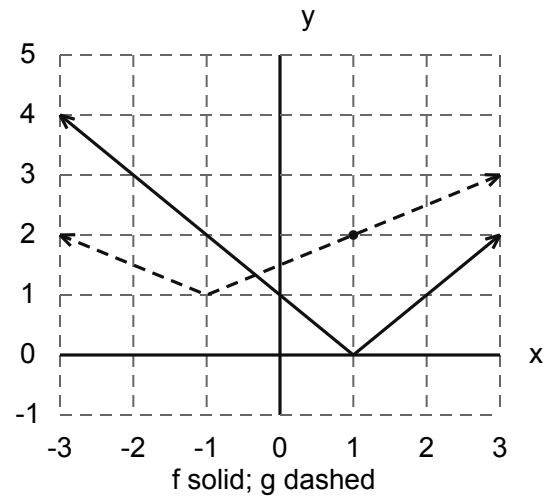
POINT CORRESPONDENCE

On f	On g
(0,0)	(-1,-1)
(1,1)	(0,1)
(2,2)	(1,3)

- 2 Using $g(x)=a f(x-h)+k$, find a,h,k from these graphs of $f(x)=x^2$ and g . Use the vertex and $g(0)=1$. Name each transformation.



- 3 $f(x)=|x-1|$. Use the fixed form $g(x)=a f(x-h)+k$, without extra horizontal scaling. Find a,h,k from the vertices and the marked grid value $g(1)=2$.



- 4 This time the form is $g(x)=f(bx)+k$. The rows are known point correspondences for $f(x)=x+2$. Find b,k and identify both changes.

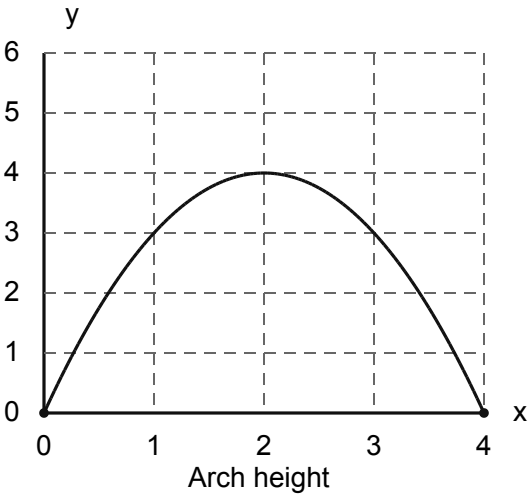
POINT CORRESPONDENCE

On f	On g
(-2,0)	(2,3)
(0,2)	(0,5)
(2,4)	(-2,7)

APPLY IT Show your work

1

An arch height is $g(x)=a f(x-h)+k$, where $f(x)=x^2$ and $0 \leq x \leq 4$ meters. Its highest point is $(2,4)$, and it meets the ground at $(0,0)$ and $(4,0)$. Use the supplied graph to find a, h, k . Explain the reflection and shifts.



2

A signal display uses $g(x)=f(bx)+k$. These known point correspondences are for $f(x)=|x-1|$. Find b, k , then describe the horizontal scale, reflection and vertical shift. Verify the third row.

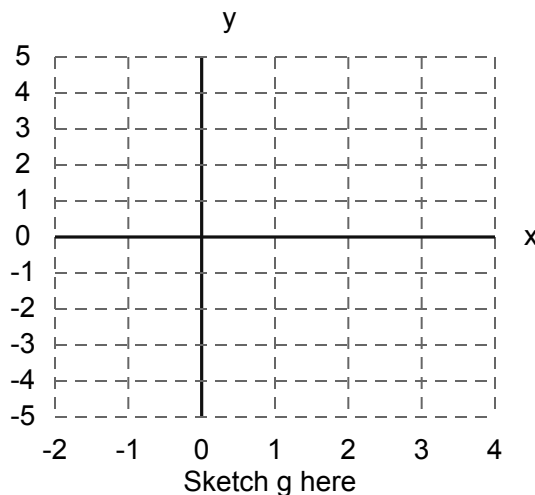
SIGNAL POINT CORRESPONDENCE

On f	On g
$(-2,3)$	$(1,4)$
$(0,1)$	$(0,2)$
$(2,1)$	$(-1,2)$

CHALLENGE One step up

A graph g is made from $f(x)=x^2$ by shifting right 1, stretching vertically by 2 and reflecting in the x -axis, THEN shifting up 3. Write g in the fixed form and sketch it. A student instead writes $-2[f(x-1)+3]$. Explain the error using both vertices.

WORK SPACE AND ANSWER



PRACTICE 1

$h=-1, k=-1, a=2$. Left 1, vertical stretch by 2, down 1. Last row: (2,2) maps to (1,3), since $2(2)-1=3$.

PRACTICE 2

Vertex (1,2) gives $h=1, k=2$. $1=a(0-1)^2+2$ gives $a=-1$. Right 1, reflection in the x-axis, up 2; $g=-f(x-1)+2$.

PRACTICE 3

The vertex moves from (1,0) to (-1,1), so $h=-2, k=1$. At $x=1$, $f(1+2)=2$, so $2=2a+1$ and $a=1/2$. Left 2, vertical compression by $1/2$, up 1.

PRACTICE 4

New input is the negative of the original, so $b=-1$. Each output rises by 3, so $k=3$. Reflection in the y-axis, then up 3: $g=f(-x)+3$.

APPLY IT 1

$h=2, k=4$; $0=a(0-2)^2+4$ gives $a=-1$. $g=-(x-2)^2+4$ on $[0,4]$: right 2, reflect in the x-axis, up 4. The vertex and one other point determine the stated form; check the second ground point.

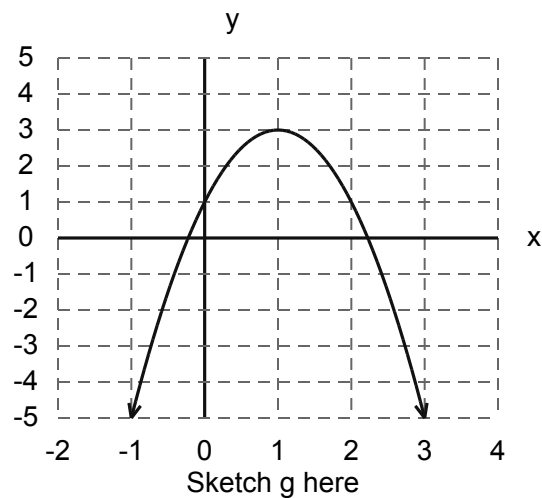
APPLY IT 2

$b=-2, k=1$: horizontal compression by $1/2$, reflection in the y-axis, up 1. For (2,1) to (-1,2), $b(-1)=2$ and $1+1=2$.

The correspondence explicitly identifies original inputs, resolving repeated-output ambiguity.

CHALLENGE

Correct: $g = -2f(x-1)+3$, vertex $(1,3)$, with $(0,1)$, $(2,1)$. The other formula shifts up before multiplying; it becomes $-2f(x-1)-6$, with vertex $(1,-6)$.



The fixed form excludes an additional horizontal scale, so the parabola parameters are identifiable. The word-problem correspondences are givens, not inferences from equal output values. Require the challenge graph as well as the formula; reflection occurs before the final vertical translation.