

Inverse Detective

Decide when reversing a function still gives a function.

HIGH SCHOOL • MA.912.F.3.6

MA.912.F.3.6 "Determine whether an inverse function exists by analyzing tables, graphs and equations."

NAME _____

DATE _____

CHECK ONE-TO-ONE BEHAVIOR

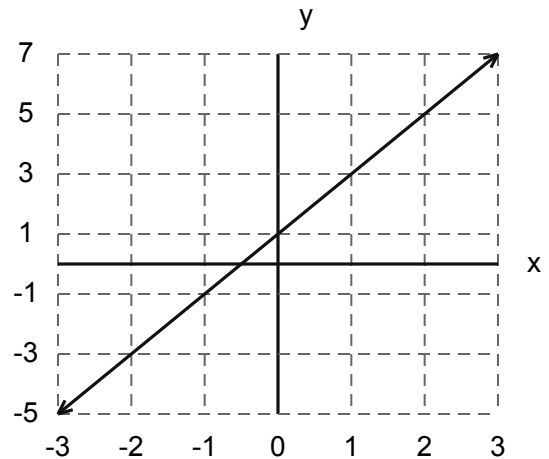
An inverse function exists exactly when each output comes from at most one allowed input. In a table, check for repeated outputs at different inputs. On a graph, every horizontal line must meet at most one point. For an equation, use algebra and its stated domain; one repeated-output example disproves one-to-one behavior. A domain restriction may restore it. The inverse swaps the original domain and range.

WORKED EXAMPLE

The graph is $f(x)=2x+1$ on all real x . Does its inverse exist, and what is it?

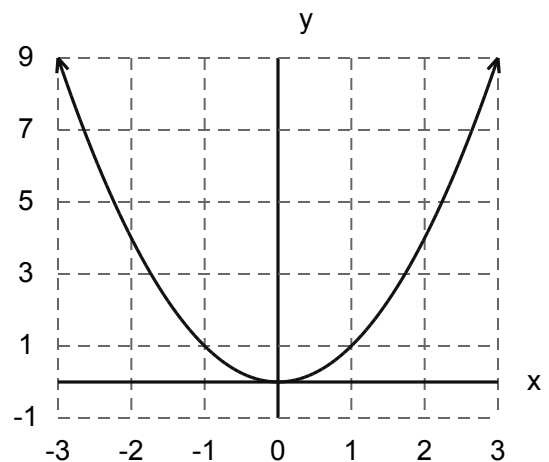
1. The line rises strictly, so no horizontal line meets it twice.
2. Solve $y=2x+1$ for x : $x=(y-1)/2$. Swap input and output names.

Answer: Yes. The inverse is $f^{-1}(x)=(x-1)/2$; its domain and range are all real numbers.



PRACTICE 4 problems

- 1** For this graph of $f(x)=x^2$ on all real x , draw the horizontal line $y=4$. Give the two intersection points. Does f have an inverse function on this domain? Explain.

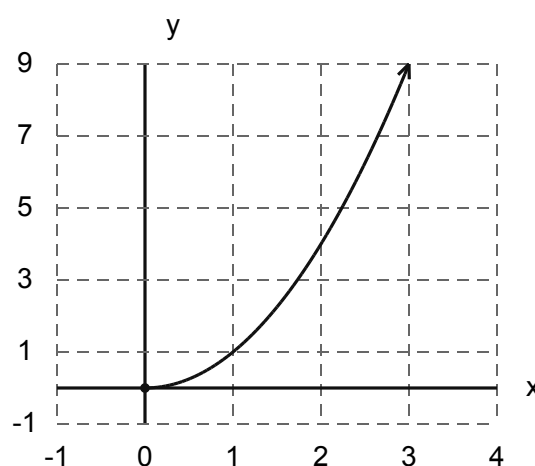


- 2 This table is the entire function, with exactly the four listed inputs. Does its inverse exist? Write the reversed pairs that justify your decision. Would removing just $x=1$ fix the issue?

COMPLETE FINITE FUNCTION

x	$f(x)$
-2	2
-1	5
1	5
3	8

- 3 Now $f(x)=x^2$ has domain $x \geq 0$, as shown by the single branch. Does an inverse exist? State its equation, domain and range. Explain why the previous counterexample is no longer allowed.



- 4 For $f(x)=x^3+1$ on all real x , prove that equal outputs force equal inputs: start with $f(a)=f(b)$. Then find the inverse formula and its domain.

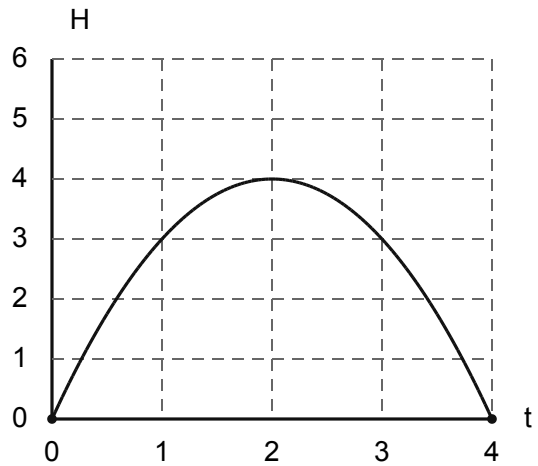
APPLY IT Show your work

1 A temperature conversion is $C(F) = \frac{5}{9}(F - 32)$ for $32 \leq F \leq 212$. The table is a sample of this stated continuous model. Can a Celsius reading uniquely recover Fahrenheit? Give the inverse equation and its allowed Celsius inputs.

CONVERSION SAMPLES

F (degrees)	C (degrees)
32	0
77	25
122	50
212	100

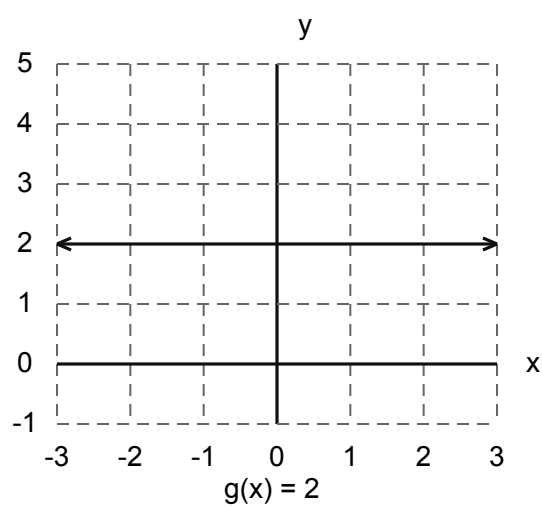
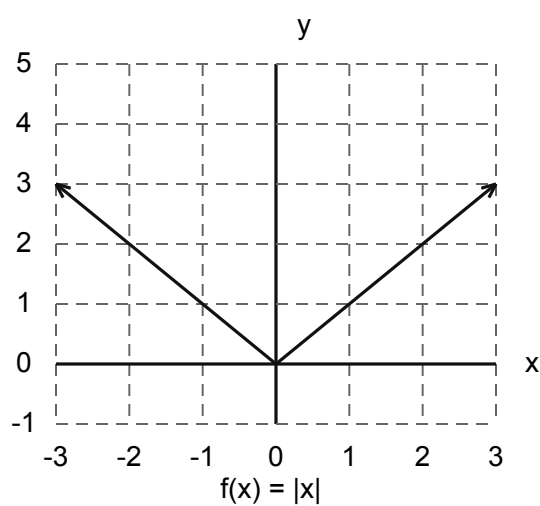
2 A toy follows height $H(t) = 4 - (t - 2)^2$ for $0 \leq t \leq 4$ seconds. Can height alone identify the time? Give a counterexample from the graph. State two different largest intervals, each with height range $[0, 4]$, on which time CAN be recovered uniquely.



CHALLENGE One step up

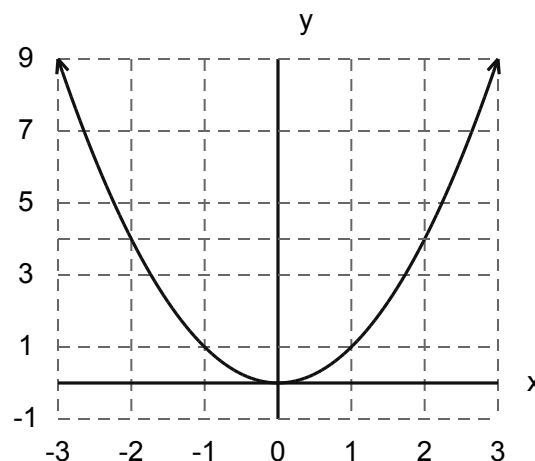
Both functions have all-real domains. Which, if either, has an inverse function? Give a specific repeated-output counterexample for each failure. Can either be repaired by restricting to a nontrivial interval (one containing more than one input)?

WORK SPACE AND ANSWER



PRACTICE 1

Intersections $(-2,4),(2,4)$. No inverse function: output 4 would need to map back to both -2 and 2.

**PRACTICE 2**

No: $(5,-1)$ and $(5,1)$ repeat inverse input 5 with different outputs. Other reversed pairs are $(2,-2),(8,3)$. Removing $x=1$ leaves distinct outputs 2,5,8, so yes.

PRACTICE 3

Yes: $f^{-1}(x)=\sqrt{x}$, with domain $[0,\infty)$ and range $[0,\infty)$. The negative input -2 is excluded; the remaining branch rises strictly.

PRACTICE 4

$a^3+1=b^3+1$ gives $a^3=b^3$, so $a=b$ over the reals. Thus an inverse exists: cube root of $(x-1)$, domain all real.

APPLY IT 1

Yes; the model is strictly increasing, not merely distinct in the sample. $F(C)=(9/5)C+32$ for $0 \leq C \leq 100$. Inverse outputs range from 32 to 212 degrees F. Reverse the linear conversion and swap the model domain and range.

APPLY IT 2

No: $H(1)=H(3)=3$. Restrict to $[0,2]$ (rising) or $[2,4]$ (falling). Each covers every height from 0 to 4 exactly once.

The two halves have opposite monotonic directions; either can supply a valid inverse.

CHALLENGE

Neither. $f=|x|$: $f(-1)=f(1)=1$; restrict to $[0,\infty)$ or $(-\infty,0]$. $g=2$: $g(0)=g(1)=2$; every nontrivial interval still repeats output 2, so none works.

Inverse means an inverse FUNCTION, not just a reversed relation. Domain restrictions must be stated. Distinct values in a finite sample do not establish one-to-one behavior of an unstated continuous extension. For the constant function, a singleton restriction works but is excluded by the challenge.